#### **Replies to Reviewer #1:**

Thank you very much for these constructive comments. We have revised the manuscript carefully based on these suggestions. We also re-calculated salinity mixing in the CESM in a more reasonable way, which can be better compared to that used in the 4-box model. Previously, we calculated  $k_{em}$  and  $k_{vm}$ . We found these coefficients could not show the influence of variation in salinity mixing on the change in vertical salinity gradient, since both the mean and anomalous vertical salinity gradients were included when calculating these coefficients. In this revised manuscript, we plot the scatter plots of anomalous salinity mixing versus anomalous vertical salinity gradient instead; we can see more clearly that the salinity anomaly is damped more effectively when the AMOC anomaly is large. The following are our point-to-point replies.

"The authors have patiently answered my questions and addressed my comments regarding the previous version of this manuscript. This version has been significantly modified, which makes a point-to-point response a little hard to interpret. The new version has replaced the concept of "two types of convection" with "enhanced vertical mixing" to stabilize the multi-centennial oscillation of the thermohaline circulation, which is further demonstrated using the results of a coupled climate model and analyses of simple box models. This version is much more clear and well-structured. I would like to suggest a new round of minor revision of this version before recommending it published."

**Responses:** Thank you very much for your invaluable suggestions, which help us improve the manuscript tremendously.

#### Major comments:

1. For the last two panels of Fig. 2, it will be more clear to plot density anomalies due to temperature and salinity changes (i.e.,  $\alpha(T,S)\Delta T$  and  $\beta(T,S)\Delta S$  instead of  $\Delta T$  and  $\Delta S$ , and use the same color range for the three panels showing density fluctuations.

Responses: Thank you very much for the suggestion. We have replotted Fig. 2 as suggested.

2. Still for Fig. 2, it may be helpful to add a panel to directly show stratification, since you discussed the response of stratification to MOC changes in lines 170-179.

**Responses**: Thank you very much for the suggestion. Since the stratification change can be easily inferred from the vertical pattern of density change in Fig. 2b, it is better not to add a subplot of stratification, to keep the figure concise.

3. Equation. 1 and lines 189-190: does the quantity  $k_v$  defined and calculated here include KPP or GM parameterizations? If the latter is used in this simulation, how do you distinguish the vertical component of GM from parameterized vertical turbulence?

**Responses**: Thank you very much for the comment.  $k_v$  includes the KPP but does not include the GM parameterization.

In this round of revision, we re-calculated salinity diffusion and eddy-induced salinity mixing using a more accurate approach. The evolution of anomalous salinity gradient (expressed as  $\Delta S'$ ) between the subpolar upper ocean (above 1000 m) and deeper ocean is plotted in Fig. 2a. It is then straightforward that a linear relationship exists between  $\Delta S'$  and AMOC variation (expressed as q'). The relationship between salinity mixing and  $\Delta S'$  is explored using Fig. 3.

Figure 3a suggests that the anomalous salinity in the subpolar upper ocean is damped more effectively when  $\Delta S'$  is large. This nonlinear feature is mainly caused by vertical salinity mixing when the AMOC is strong (i.e.,  $\Delta S' > 0$ , q' > 0) (Fig. 3c), and by both horizontal and vertical salinity mixing when the AMOC is weak (i.e.,  $\Delta S' < 0$ , q' < 0) (Fig. 3b). This nonlinear feature can be physically understood as follows. When the stratification in the subpolar ocean is weak and the AMOC is strong, upper-ocean salinity anomaly can be mixed into the deeper ocean more efficiently by salinity diffusion process. When the stratification in the subpolar ocean is strong and the AMOC is weak, salinity diffusion is weak but eddy-induced salinity mixing can enhance the downward mixing. Instead of canceling each other completely, the combined effect of diffusion and eddy-induced mixing is that salinity mixing is enhanced when the AMOC anomaly is large. In the revised manuscript, we use "enhanced mixing" for "enhanced eddy-induced mixing or diffusion when the AMOC anomaly is large" for simplicity.

Section 2 is revised substantially. More detail can be found in lines 191-226.

4. Lines 193-203: You seem to imply that a process is useful only when it is strong. However, a weakening process can also make a difference. Such statements are better made after the salinity/density budget is analyzed. You showed the budget for your box model. It may also be helpful to show the same for the CESM model, if possible. For me, the important message after

this analysis is that there is always some "vertical stirring processes" that are strong no matter the MOC is strong or weak, let it be either mixing or diffusion. This is what you show to play an essential role in the self-sustained oscillation in your box model.

**Responses**: Thank you very much for the suggestions. In this work, we focus on simple box model studies. We are working on detailed analyses on the CESM model results, and will submit a manuscript on multi-centennial variability in the CESM model in the near future.

There are two more reasons for not showing the CESM budget in this manuscript. First, the variations of vertical diffusion and eddy-induced vertical mixing are very small (similar to those shown in Fig. 6c from the box model), which may give a false impression to the readers that vertical mixing is not important. Second, based on the CESM model results, it is not easy or accurate to separate the anomalous flux into salinity-driven flux or velocity-driven flux; so the CESM budget cannot be shown in the same way as for the box model. As for the "the important message," it is exactly what we want to show, and we have revised related statements in the manuscript.

In addition, you mention that "a weakening process can also make a difference." We agree with you on this completely. We are diagnosing the CESM results intensely; and hopefully, we can understand better "how a weakening process affects the AMOC and makes differences physically."

# 5. Lines 273-277: I do not really understand the need to plot the terms timed by S02 instead of the terms in your box model equations directly.

**Responses**: Thank you very much for the comment. We did this in order to see more clearly the positive and negative contributions to the amplitude.

In Fig. 6c, after multiplied by  $S_2^{\prime}$ , even they are still varying periodically, we can clearly see the long-term mean of the perturbation advection of mean salinity is positive (green curve), helping the growth of the oscillation, while that of mean advection of anomalous salinity is negative (red curve), tending to suppress the growth. However, if we do not multiply them by  $S_2^{\prime}$ , the long-term mean of these terms is zero. We modified related statements to show this intention better in the revised manuscript.

6. Lines 285-286 and eq. 8: (1) Explain in a bit more detail what you are doing here, such as: at a given strength of AMOC, you take the stronger one out of mixing or diffusion, and make a quadratic fit. (2) The scatter of the green dots from the fit is not as ideal as the gray dots. You may want to further justify your choice of a quadratic fit.

**Responses**: Thank you very much for the comment. We have added more detail in the revised manuscript. Please refer to our response to Question #3.

In lines 191-226 of the revised manuscript, we emphasize the key point, namely, salinity mixing should be strong enough after disturbance grows, and the quadratic fit is not that critical. However, the new Fig. 3 does not suggest cubic regression between salinity mixing and  $\Delta S'$  (red curve in Fig. 3a). After considering the linear relationship between  $\Delta S'$  and q', we think the enhanced mixing effect can be roughly expressed as follows,

$$F_{mixing} \sim (\Delta S')^3 \sim (q')^2 \Delta S' \sim k_m \Delta S'$$

where  $k_m \sim (q')^2$ , parameterizing the enhanced mixing effect when the AMOC is large. Here, we would like to emphasize that the exact form of the above equation and  $k_m$  are not important. The regression between salinity mixing and  $\Delta S'$  can be linear or even the power of five. The cubic relationship is just a reasonable outcome from the coupled model we used in this work. This enhanced mixing mechanism is then used in the 4-box model in Section 4.

7. It is well-known that vertical mixing is poorly quantified in the real ocean. In addition to how the behavior of the system depends on the volume ratios, as already explored very well in this manuscript, it will also be interesting and relevant, to explore how the magnitude of this enhanced mixing influence the behavior of the system. It can be imagined that, based on the difference between the 4-box and 3-box models, there should be a critical k, that determines the regime where self-sustained oscillation exists or not.

**Responses**: Thank you very much for the comment. We have examined carefully how the magnitude of enhanced mixing affects the behavior of the system, and we also found the critical  $k_m$ .

The critical  $k_m$  is shown in Fig. R1. The necessary and sufficient condition for the existence of a self-sustained oscillation is that  $k_m$  is smaller than the critical value at equilibrium. And the critical value will become larger when the AMOC is strong or weak. The specific form of  $k_m$  is not important. This is way we say "A background vertical mixing can also be included, say,  $k_m = \overline{k}_m + k'_m$ , which will not qualitatively change the results as long as  $\overline{k}_m$  is not too big."



Figure R1 The critical  $k_m$  for different M using the parameters in Table 1. Self-sustained oscillation cannot exist if  $k_m$  is larger than this critical value at equilibrium.

- 8. Line 25: "which showed that the vertical mixing or diffusion. . . " Revised.
- 9. Line 46: replace "These variabilities exist in" with "Multi-centennial variabilities have been observed in". Revised.
- 10. Line 53: replace "concern" with "focus". Revised.
- Line 74: add "a better understanding and representation of the" before the word "multicentennial". Revised.
- 12. *Line 75: replace "provide a source of" with "help improve the".* Revised.
- 13. Line 78: "models of different complexities". Revised.
- 14. *Line 93: "the period of THC oscillation".* Revised.
- 15. Line 101: replace "water" with "ocean". Revised.
- 16. Line 102: add "that" after "assuming". Revised.
- 17. Line 104: replace "achieved" with "realized". Revised.
- 18. Line 136: "Analyses of". Revised.
- 19. Line 138: remove "by". Revised.
- 20. Line 142-146: there is some repetition between these sentences. Try to combine them. Revised.
- 21. Line 146: "each velocity component". Revised.
- 22. *Line 150: "3000 m depth"*. Revised.

23. Line 151-152: the time-scale looks like 200-300 years to me instead of 400 years as stated in the manuscript. Can you verify this number, maybe via a spectrum analysis?

**Responses**: Thank you very much for the comment. The spectrum is shown in Fig. R2c, which indicates the timescale longer than 300 years and slightly shorter than 400 years.



Figure R2 The AMOC index and its spectrum. The ordinates of (b) and (c) are normalized spectrums, while the abscissas are frequency and period, respectively.

- 24. *Lines* 165-166: *This sentence implies causality, inconsistent with the next sentence.* Revised.
- 25. Lines 178-179: this sentence is a repetition of previous statements, and can be removed.

**Responses**: Thanks for the suggestion. Removed.

26. *Line 191: specify the area over which both \_ quantities are calculated.* 

**Responses**: Thank you very much for the suggestion. The region is specified in Fig. 3d.

- 27. *Lines 196-197: how can you be sure of this causality?* This part is totally rewritten.
- 28. *Line 213: "and have been used".* Revised.
- 29. Line 228: "transports" instead of "transport". Revised.
- 30. *Line* 269: "perturbed" instead of "perturbation". Revised.
- 31. Line 290: "vertical mixing or diffusion".

**Responses**: It has been stated in Section 2, for the sake of concision, we will use "enhanced mixing" for "enhanced eddy-induced mixing or diffusion when the AMOC anomaly is large."

- 32. Line 302: "remove upper ocean fresh water downward to lower ocean" can be replaced with "mix the anomalous freshening signal in the upper ocean downward". Revised.
- 33. *Line 307: "to stabilize" instead of "in stabilizing".* Revised.
- 34. *Line 313: replace "would eventually become" with "almost instantly becomes".* Revised.
- 35. *Line 314: remove "at this particular instant"*. Revised.
- 36. Line 343: replace "oscillation" with "oscillatory". Revised.
- 37. Line 372-373: replace "would also spend more time" with "also spends more time". Revised.
- Line 375: replace "that in Sevellec et al. (2006), which showed . . " with "the results of Sevellec et al. (2006) that". Revised.
- 39. Line 377: replace "more possibilities" with "a higher probability". Revised.
- 40. *Line 402: replace "organized" with "constrained".* This part is rewritten.
- 41. *Line 412: replace "owed out" with "removed".* Revised.
- 42. Line 413: replace "since the 3-box model is stable" with ", similar to the 3-box model as a limit". Revised.
- 43. *Line 429: "and without"*. Revised.
- 44. Line 449: replace "less observed" with "under-appreciated". Revised.
- 45. *Line 474: replace "instinct" with "intrinsic", and add ", regardless of external forcing" at the end of this sentence.* Revised.
- 46. *Line* 490: "positive salinity advection feedback". Revised.
- 47. Line 491: replace "eigenmode" with "eigenvalue". Revised.
- 48. Lines 493-494: Dosen't the 3-box model implicitly include enhanced vertical mixing in the subpolar region already?

**Responses**: We removed the statement. Yet, the implicit mixing in the 3-box model should not be seen as "enhanced mixing," since the weak mixing around equilibrium is not considered in the 3-box model.

49. *Line 500: "extreme vertical mixing event"*. Revised.

- 50. Line 502: replace "their" with "these two". Revised.
- 51. Line 504: replace "too much" with "and lead to an unbounded growth". Revised.
- 52. Line 507: "density gradient". Replace "consideration" with "assumption". Revised.
- 53. Line 511: replace "differ, but they are not contradictory" with "are not contradictory, although different". Revised.
- 54. Line 516: replace "drawing a conclusion with" with "determined". Revised.
- 55. *Line* 525: "a more realistic but still simple model". Revised.
- 56. Line 527: replace "different models with different complexities" with "a hierarchy of models of different complexities". Revised.
- 57. Line 529: add "and interpretations of the on-going and future climate change" to the end of this sentence. Revised.

#### **Replies to Reviewer #3:**

Thank you very much for these constructive comments. We have revised the manuscript carefully based on these suggestions. Ws re-calculated the salinity mixing in the CESM in a more reasonable way, which can be better compared to that used in the 4-box model. Previously, we calculated  $k_{em}$  and  $k_{vm}$ . We found these coefficients could not show the influence of variation in salinity mixing on the change in vertical salinity gradient, since both the mean and anomalous vertical salinity gradients were included when calculating these coefficients. In this revised manuscript, we plot the scatter plots of anomalous salinity mixing versus anomalous vertical salinity gradient instead; we can see more clearly that the salinity anomaly is damped more effectively when the AMOC anomaly is large. The following are our point-to-point replies.

This manuscript describes an idealized model of centennial variation of the Atlantic Meridional Overturning. The study suggests the need for "mixing" (by either small vertical turbulence or mesoscale eddy turbulence) in the subpolar region to maintain a self-sustained variability.

This study tackles an important and interesting topic of today climate science. I feel that the changes in the manuscript have strongly improved the manuscript.

I do recommend this work for publication after minor corrections.

**Responses:** Thank you very much for your invaluable suggestions, which help us improve the manuscript tremendously.

### **Specific Comments:**

1. 1.50-51: In this context I suggest to use Atlantic Meridional Overturning Circulation (AMOC)

Responses: Thank you very much for the suggestion. All "THC" is changed to "AMOC."

2. *l.56-58: To the best of my knowledge this is rather the "internal" variability that dominates at decadal timescale. You should change "decadal" to "multi-decadal".* 

Responses: Thank you very much for the suggestion. Revised.

3. *l.*67-76: In this context, it seems appropriate to cite Bonnet R., et al. (2021).

Responses: Thank you very much for the suggestion. The paper is cited.

- 4. 1.98, 1.121, and 1.500: Please change "Colin De Verdière" to "Colin de Verdière". Revised.
- 5. *l.130: Is "exhibited" better than "realized"?* Revised.

6. *l.133:* Could you give the resolution of the oceanic model? (This is a quite important information to understand the nature of the  $\overline{w'S'}$  term used later.)

**Responses**: Thank you very much for the suggestion. The horizontal ocean grid has a uniform  $1.125^{\circ}$  spacing in the zonal direction, and non-uniform spacing in the meridional direction, which resolution is  $0.27^{\circ}$  near the equator, extending to the maximum  $0.65^{\circ}$  at  $60^{\circ}$ N/S and then shrinking gradually to the poles. In this revised manuscript, the  $\overline{w'S'}$  effect is not calculated directly; instead, it is calculated as the residual. Detailed statement can be found in lines 191-226.

- 7. *l.165: Please replace "would weaken" by "is concomitant"* Revised.
- 8. *l.166: Please replace "can act to enhance the AMOC" by "is also weak".* Revised.
- 9. *l.178: Please replace "damping" by "compensating"* Revised.
- 10. *l.187* and equation (1): Whereas the kvm computation seems to naturally holds well, since it is "just" a large scale parametrization of a diffusive process, the kem is more problematic. Here you assumed that the small scale advection can be represented with a Fick law (down-gradient flux of salinity). This will not conserved gradients, unlike GM assumption (which rotates but conserves the ispoyen and is more often used to represent eddy-induced effect). A word on the limit of this assumption would be nice.

**Responses**: Thank you very much for the suggestion. This part is revised significantly. We recalculated salinity mixing and gave up  $k_{em}$  or  $k_{vm}$ . Figure 3 is replotted, showing the relationship between salinity mixing and anomalous salinity gradient (expressed as  $\Delta S'$ ) between the subpolar upper ocean (above 1000 m) and deeper ocean.

Figure 3a suggests that the anomalous salinity in the subpolar upper ocean is damped more effectively when  $\Delta S'$  is large. This nonlinear feature is mainly caused by vertical salinity mixing when the AMOC is strong (i.e.,  $\Delta S' > 0$ , q' > 0) (Fig. 3c), and by both horizontal and vertical salinity mixing when the AMOC is weak (i.e.,  $\Delta S' < 0$ , q' < 0) (Fig. 3b). This nonlinear feature can be physically understood as follows. When the stratification in the subpolar ocean is weak and the AMOC is strong, upper-ocean salinity anomaly can be mixed into the deeper ocean more efficiently by salinity diffusion process. When the stratification in the subpolar ocean is strong and the AMOC is weak, salinity diffusion is weak but eddy-induced salinity mixing can enhance the downward mixing. Instead of canceling each other completely, the combined effect of diffusion and eddy-induced mixing is that salinity mixing is enhanced when the AMOC anomaly is large.

Section 2 is revised substantially. More detail can be found in lines 191-226.

11. 1.189 and the use of  $\overline{w'S'}$ : Could you be more explicit about this term. What "w" stands for? Eulerian, Eddy-induced, and residual vertical velocity? What do the bar and ' refer to? A subpolar spatial average and its anomaly? I feel that  $\overline{w'S'}$  is the eddy-induced salinity transport (from GM), but I can only guess...

**Responses**: Thank you very much for the comment. We use  $\overline{w'S'}$  for the eddy-induced salinity transport, where the prime is time deviation from the equilibrium, and the bar refers to the average over a grid cell. It is called eddy-induced vertical salinity mixing. Original Eq. (1) is removed. Please see the text in lines 191-226.

12. *l.197: Please remove "acting to enhance the AMOC". You have not demonstrated this causal link.* 

Responses: Thanks for the suggestion. This paragraph is totally rewritten.

13. l.211: The choice of 500 m seems odd. The flow inversion in the subtropics is around 1,000-m depth (according to your Fig. 1, as well as observations, e.g., Rapid data). It seems that 1,000-m depth would have been better choice for the box limit.

**Responses**: Thank you very much for the suggestion. Using the CESM outputs, the vertical salinity mixing within the upper 1000-m depth is calculated and shown in Fig. 3. For the box model, the upper ocean is still 500 m in thickness, to make the upper-ocean turnover time in the box model similar to that in the CESM, considering  $\bar{q} = 10 Sv$  in the box model, which is much smaller than that in the CESM.

14. l.271-272 and Fig. 5b: This is circulation reversed earlier... Again there is no sense in comparing q' and q. The set of equations (6) are not design for a comparison with q, since they can be rescaled at will. Please plot only q' (e.g., Fig. 5b and 6b) and stop discussing q' in relation to q.

Responses: Thank you very much for the suggestion. Figures 5, 6 and 11 are replotted as suggested.

15. *l.273:* This is not correct. The linear approach does not allow you to conclude that. You might want to say: "If the linear framework holds for perturbation as big as the mean, the THC would shutdown in several thousand years." However this does not make a lot of sense and so it is kind of useless...

Responses: Thank you very much for the suggestion. This sentence is deleted.

- 16. 1.276-277: You should cite Marotzke (1996). It is cited.
- 17. *l.452* and section 5.2: What would happen in the regime of sustained variability when you add the noise? Does the spectrum remain the same? If not could it be used to track in observations if the variability signature come from a damped oscillator or a sustained oscillation?

**Responses**: Thank you very much for the suggestion. The spectrum in the regime of sustained variability under noise is shown below in Fig. R3 (it was Fig. 8 in the first draft). The peak is narrower compared to that in the mean spectrum in the regime of damped oscillation (Fig. 10 in this revised manuscript). However, a narrow peak can also appear in the regime of damped oscillation for a single time series. Especially when the length of time is limited, the spectrum is sensitive to both initial conditions and the spectrum of noise.



Figure R3 Statistics of the AMOC forced by white noise freshwater perturbation in box 2 in the 4-box model. (e) is the annual mean AMOC index under the noise forcing. (f) is probability distribution function (PDF) for annual mean AMOC. (g)-(h) show autocorrelation and power spectrum of annual mean AMOC index, respectively. The peak period in (h) denotes the period between 300 and 400 years.

18. *l.455: Please add the unit to \lambda value.* Revised.

19. 1.458 and equation (18): This is not a SDE. Would one need an SDE to incorporate such noise?
(With a Wiener process this should take the form of dSi = [] dt + [] dWi.)

**Responses**: Thank you very much for the comment. In this work, red noise instead of white noise is used in freshwater flux. Using white noise will not change the peak in Fig. 10d, as we have shown the peak is not sensitive to  $\alpha$  in Eq. (21), and white noise can be generated when  $\alpha = 0$ . The red noise is generated in the same way as defined in Gilman et al. (1963), and sufficient information is provided for other researchers to repeat the integration.

20. 1.477-479: We need more than that... Is it a perturbation around the same mean state but with nonlinear dynamics (vs linearized one)? I suspect that it is actually quite different from equation 2 that cannot be solved by itself without parameterizing at least a  $\overline{q}$ . All these need to be addressed and clarified.

**Responses**: Thank you very much for the comment. In Table 1,  $\bar{q} = 10$  Sv, and we stated that the integration is done "using the same parameters listed in Table 1." The equilibrium state of the box model is given when introducing the 4-box model, which is the same as that of the 3-box model. We add more statements in the revised manuscript.

21. *l.486 and section 6: You suggest the role of turbulence (eddy transport). I think it is a key point that should be more highlighted (in the conclusion, the main text, and the abstract). Indeed the role of turbulence for the large-scale, climatically-relevant variability is a current "hot" topic of climate science.* 

**Responses**: Thank you very much for the comment. We add more statements as suggested.

22. *l.500: Please replace "step" by "time-step".* Revised.

23. 1.503-504: Not really, since it is only apply to the two subpolar boxes. Please clarified.

**Responses**: Thanks for the comment. Here, we mean that mixing can transport salinity anomalies out of "subpolar surface ocean," not "subpolar ocean." In this revision, the "subpolar surface ocean" is replaced by "subpolar upper ocean."

24. Fig.3: Maybe a quadratic regression and an estimated R2 (% of variance explained) would be needed. You need to be more quantitative, since it is a key point. (The "hand-drawn" curve is not really acceptable.)

**Responses**: Thank you very much for the comment. Results from cubic regression are plotted in the new Fig. 3, showing mixing is enhanced when salinity difference anomaly is large. As we stressed in the main text, these results are similar for different forms of  $k_m$ , as long as the enhanced mixing mechanism can be explicitly expressed.

The detailed expression for cubic regression in Fig. 3a is

$$F_{mixing} = -285.15 * (\Delta S')^3 - 2.51 * (\Delta S')^2 - 1.28 * (\Delta S') - 0.003$$
(R1)

The R2 is about 25%. The effective rank for the scaled Vandermonde coefficient matrix is 4, suggesting the cubic regression polynomial is significant. Therefore, after considering the linear relationship between  $\Delta S'$  and q', Eq. (R1) can be roughly expressed as follows,

$$F_{mixing} \sim -(\Delta S')^3 \sim -(q')^2 \Delta S' \sim -k_m \Delta S' \tag{R2}$$

where  $k_m \sim (q')^2$ , parameterizing the enhanced mixing effect when the AMOC is large. Here, we would like to emphasize that the exact form of the above equation and  $k_m$  are not important. The regression between salinity mixing and  $\Delta S'$  can be linear or even the power of five. The cubic relationship is just a reasonable outcome from the coupled model we used in this work. This enhanced mixing mechanism is then used in the 4-box model in Section 4.

25. Fig.8: I have the feeling that it would be more interesting to show the three regimes (as in Fig. 9, and in the text).  $\lambda < \lambda'_C$ ,  $\lambda'_C < \lambda < \lambda_C$  and  $\lambda > \lambda_C$ . The neutral values are kind of obvious ones. I imagine that you will have: \*) For  $\lambda < \lambda'_C$ , all decaying; \*) For  $\lambda'_C < \lambda < \lambda_C$  3-box decaying, 4 box growing, and 4-box-with-km self-sustained; \*) For  $\lambda > \lambda_C$ , all growing;

Responses: Thank you very much for the suggestion. We have revised Fig. 8 as suggested.

26. Fig.9 and 1.725-726: I am not sure it is an accurate description. do you mean : "For the 4-boxwith-km the centennial oscillations is decaying in region 2, self-sustained in region 1, and unstable in region 3." Also I would have labelled the regions 1, 2, and 3 with increasing M rather than 2, 1, and 3.

Responses: Thank you very much for the comment. We have revised Fig. 9 as suggested.

27. Fig.10: Please plot the spectrum with frequency (so it is related to variance - i.e., power spectral density). You could indicate the period with a second label on top if needed. Please specify that the timescales are the damping timescales of the noise in the caption.

The use of the term "signal-to-noise ratio" is odd... If I understand it is: THC-response/FWinput. This is not really a signal-to-noise ratio (since both are noises), but some kind of amplification/efficiency response... But, since it relates two variables that have different physical meaning, this is odd. Please clarify in the caption. (I haven't found its mathematical definition in the main text. Please write the expression of the signal-to-noise ratio in the main text.)

**Responses**: Thank you very much for the comment. The spectrum is re-plotted with frequency. Now, we call it "the ratio of the AMOC-spectrum to the freshwater-spectrum." In the text, "which suggests the AMOC responses most efficiently to noise whose period is about 330 years" is added to clarify the intention.

## References:

Bonnet R., et al., 2021: Increased risk of near term global warming level due to a recent AMOC weakening. Nature Communications, 12, Article number: 6108. https://doi.org/10.1038/s41467-021-26370-0

Marotzke, J., 1996: Analysis of thermohaline feedbacks, in Decadal Climate Variability: Dynamics and Predictability, D. L. T. Anderson and J. Willebrand, eds., NATO ASI Series, Series I, 44, 333-378.

Gilman, D. L., Fuglister, F. J., and Mitchell, J. M., Jr., 1963: On the Power Spectrum of "Red Noise". Journal of Atmospheric Sciences, 20(2), 182-184.