

# Self-Sustained Multi-Centennial Oscillation of Atlantic Thermohaline Circulation

YANG Haijun (杨海军)<sup>1,2</sup>, LI Yang (李洋)<sup>2</sup> and SHI Jiaqi (石佳琪)<sup>2</sup>

<sup>1</sup>Department of Atmospheric and Oceanic Sciences, Fudan University

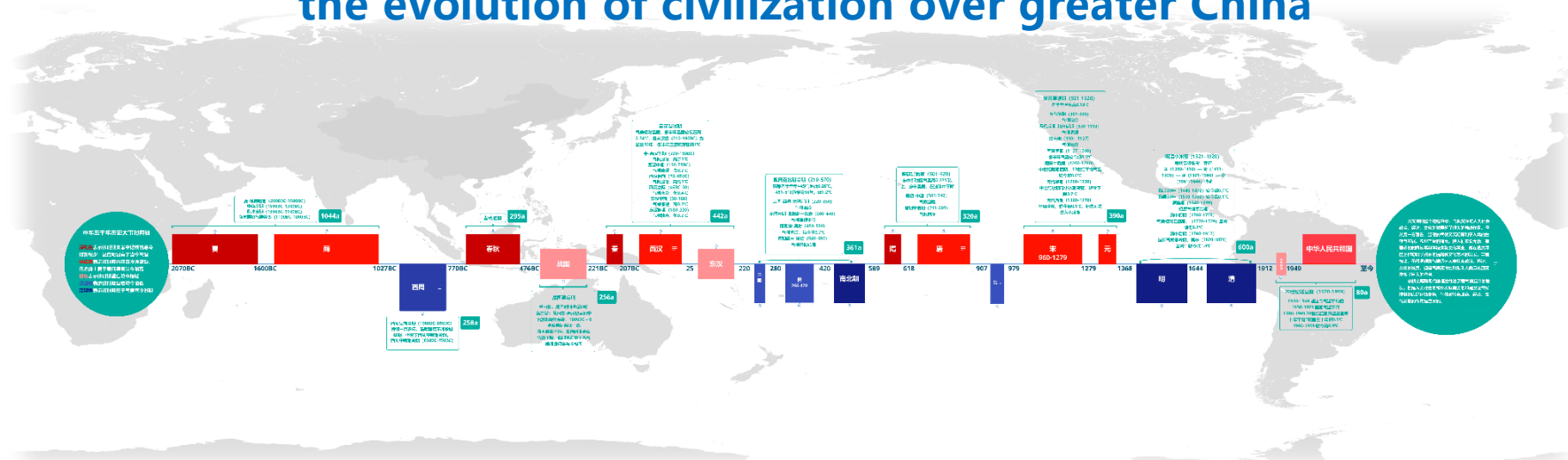
<sup>2</sup>LaCOAS and Department of Atmospheric and Oceanic Sciences  
School of Physics, Peking University

Email: [yanghj@fudan.edu.cn](mailto:yanghj@fudan.edu.cn)



# Multi-Centennial Variability: 200-300 (?) Years

## Warm and Cold period during the past 5000 years in the evolution of civilization over greater China



# Our Questions

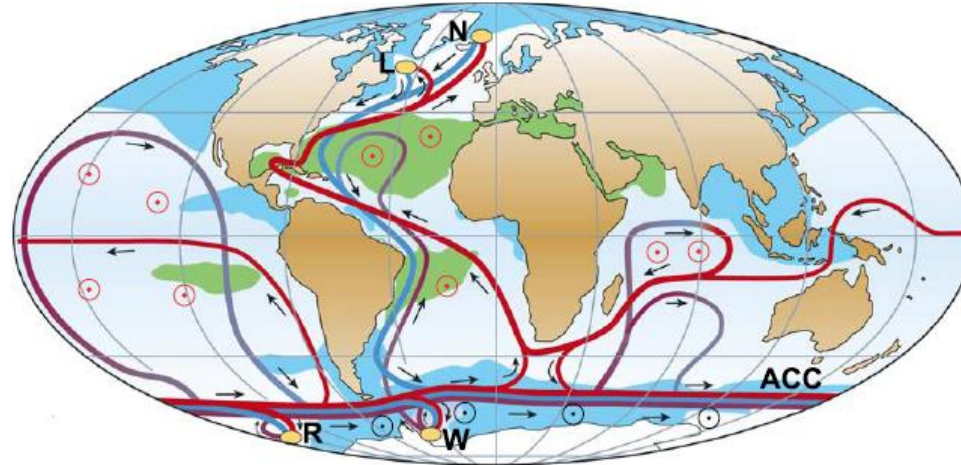
## 500±300 Years

1. *Natural Centennial-Millennial oscillation* in climate system?  
地球气候系统是否存在百年-千年尺度自然振荡?
2. Connection to the *evolution of human civilization*?  
这种振荡与人类文明演化是否有关系?

# Contents

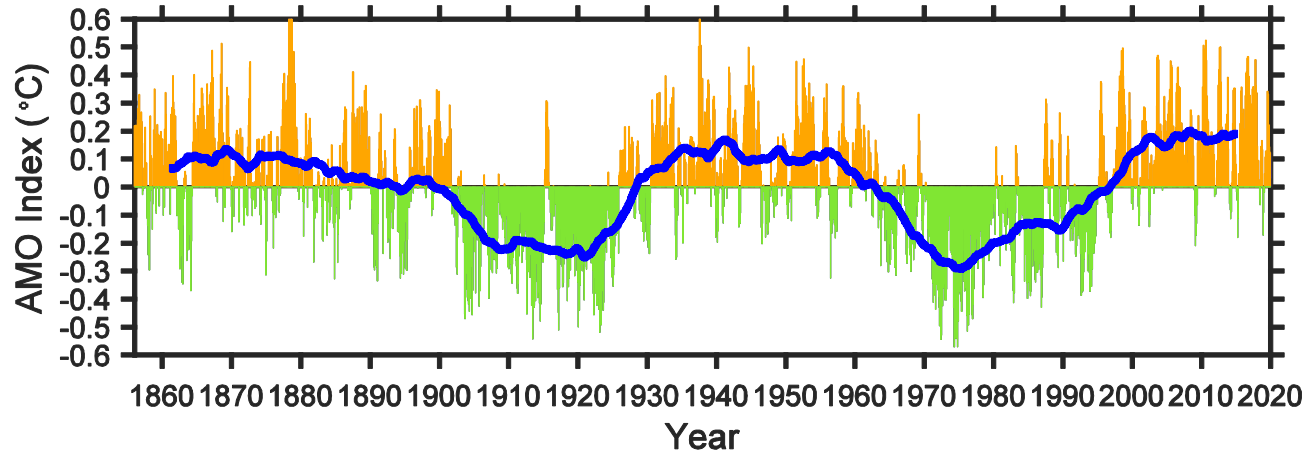
1. **Common Knowledge**
2. Motivation
3. Observation
4. Theory
5. Modeling

# Great Conveyor Belt: *Thousands'* Years



- |                      |                         |                       |
|----------------------|-------------------------|-----------------------|
| Surface flow         | Wind-driven upwelling   | <b>L</b> Labrador Sea |
| Deep flow            | Mixing-driven upwelling | <b>N</b> Nordic Seas  |
| Bottom flow          | Salinity > 36 ‰         | <b>W</b> Weddell Sea  |
| Deep Water Formation | Salinity < 34 ‰         | <b>R</b> Ross Sea     |

# AMO: 60-80 Years



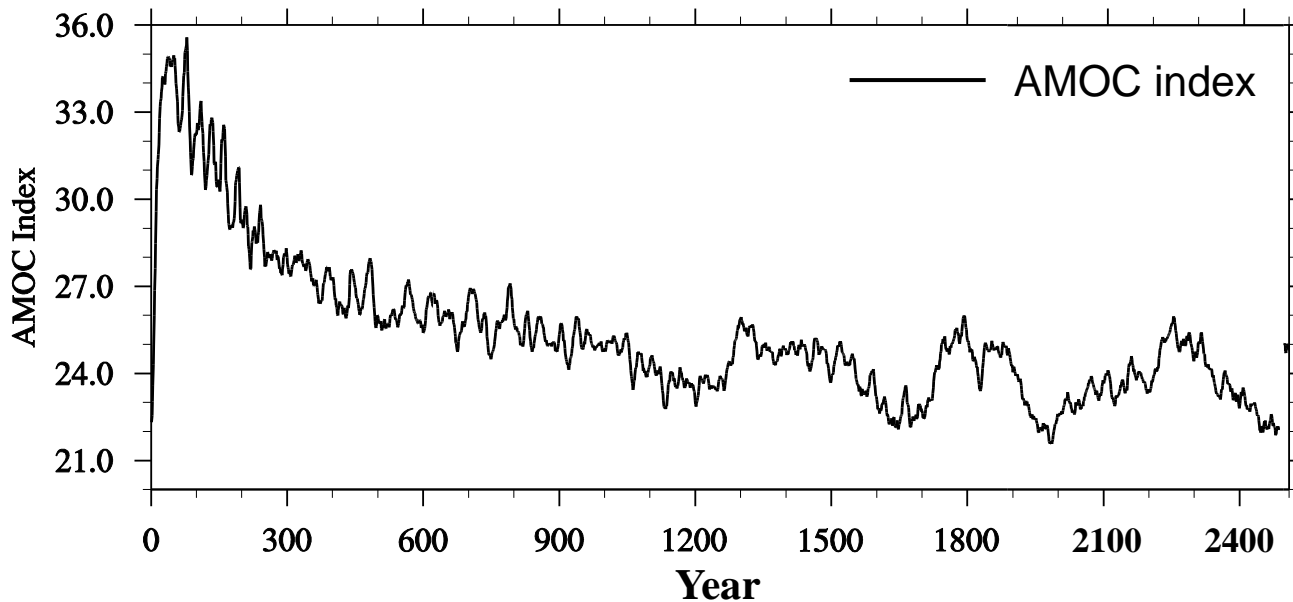
Kaplan SST (Kaplan et al., 1998; Drinkwater et al., 2014)

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# Motivation

Earlier in **2013**, we confused ...

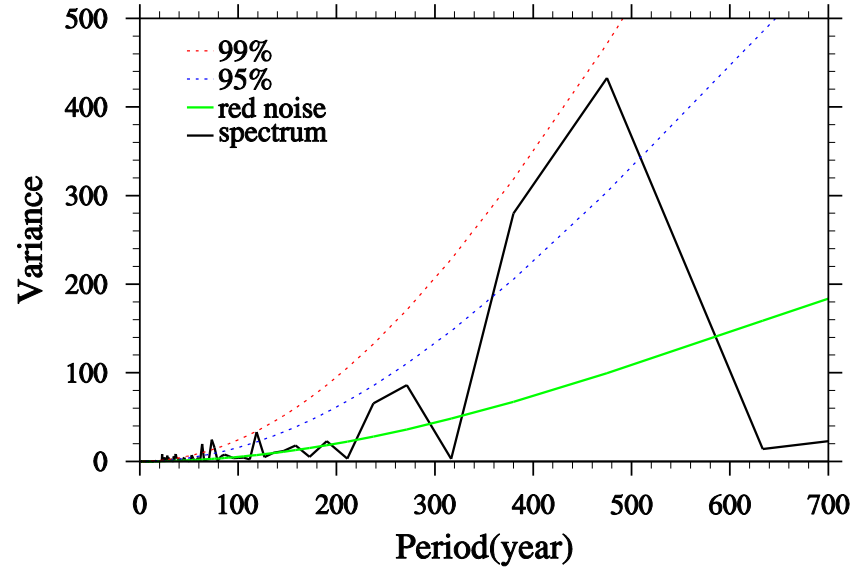
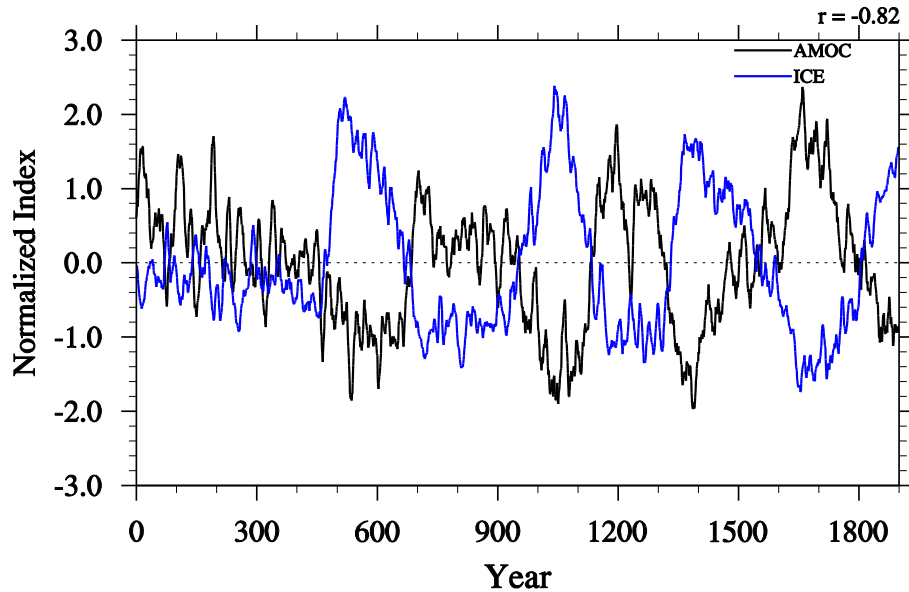


**2500** years control run using **NCAR-CESM1.0**



# Sea Ice → AMOC ?

Excellent correlation, but *causality?*

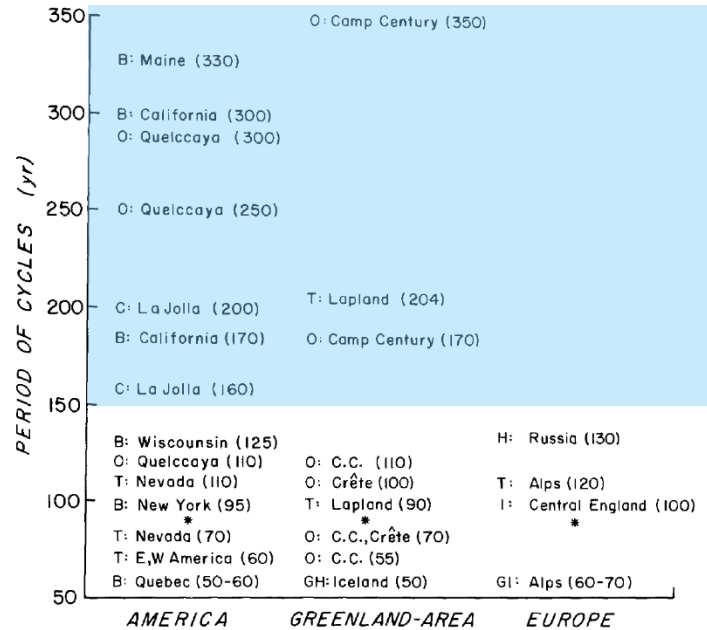
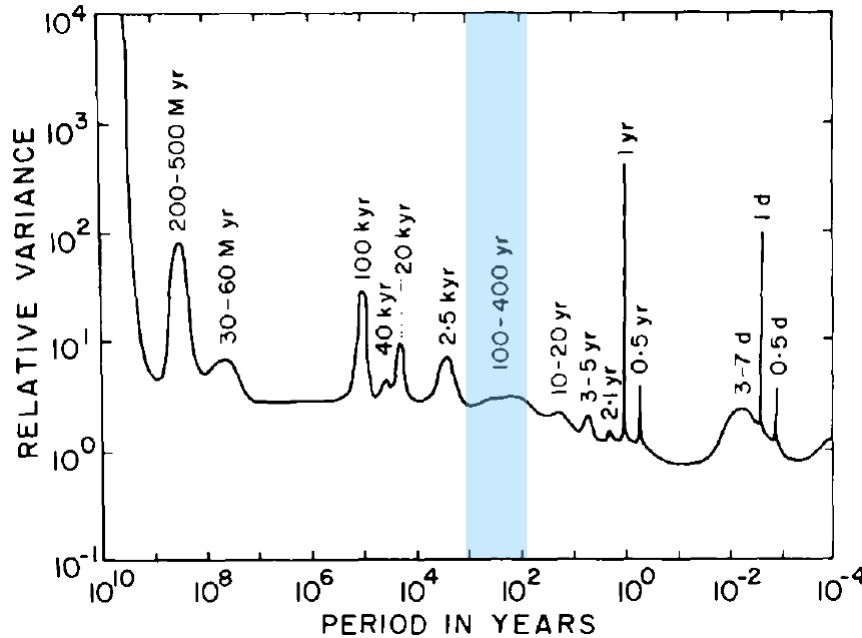


Confused ...

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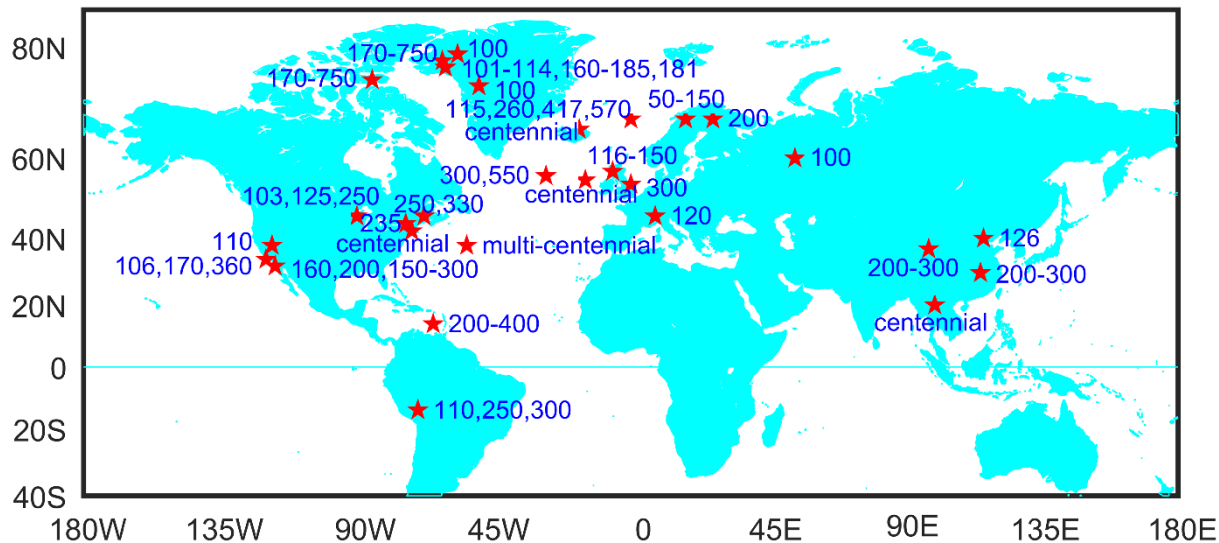
# Centennial Variability in *Proxy* Data



Stocker and Mysak (1992): Climatic fluctuations on the century time scale: A review of high-resolution proxy data and possible mechanisms. *Climate Change*.



# Centennial Variability: 200-300 (?) Years



石佳琪, 杨海军, 2021: 关于观测的百年到千年时间尺度气候振荡。科学通报, 待投稿。

# Centennial Variability: 200-300 (?) Years

文献出处	位置	代用指标	周期(年)
Siren et al., 1961 Lamb et al., 1977	拉普兰德	树轮	70,90,200
Soutar and Isaacs., 1969	加利福尼亚	海底沉积物	106,170,360
Johnsen and Dansgaard, 1970	格陵兰	冰芯	53-56,69-73,104- 144,160-185
LaMarche et al., 1974	美国内华达州	树轮宽度	70,110
Schweingruber et al., 1976	瑞士	树轮宽度	30,120
Lamb et al., 1977	英国, 俄罗斯	冬天严寒程度	300, 100
Nefel et al., 1981 Sonett et al., 1984	加利福尼亚	树轮中的 C 放射	150-300, 160,200
Fisher et al., 1982	格陵兰- 加拿大	冰芯	170-185,300-330,147- 435,625-714
Hameed et al., 1983	中国北京	降水记录	56,84,126
Thompson et al., 1989	秘鲁安第斯山脉	冰芯	110,250
Gajewski, 1988	Hells Kitchen 湖(美 国威斯康辛州)	湖底沉积物中的花粉	90-120, 230-250
Stuiver and Braziunas., 1989	/	树轮中的 14C	45,52,67, 143,218,420
Rothlisberger et al., 1989	/	树轮和冰川振荡	88,102-104,123-143

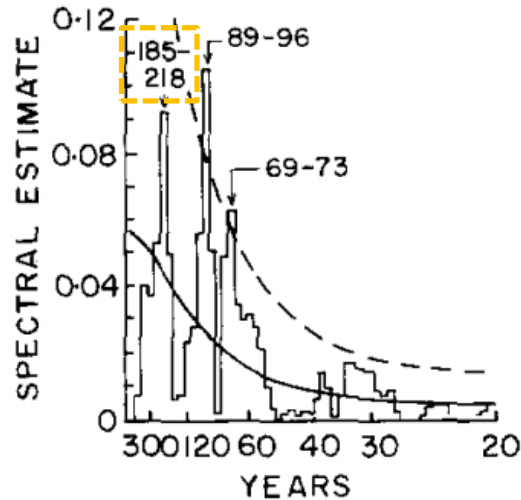
文献出处	位置	代用指标	周期(年)
Briffa et al., 1990	Fennoschandia	树轮	50-150
Anklin et al., 1998	格陵兰岛	冰川雪和冰芯	100,200
Chapman and Shackleton., 2000	北大西洋	深海沉积物	550
McDermott et al., 2001	爱尔兰西南部	/	78,169,625
Proctor et al., 2002	苏格兰西北部	石笋	72-96,116-150
Nyberg et al., 2002	加勒比东北部	有孔虫	200-400
Risebrobakken et al., 2003	挪威海	岩芯	80-115,260, 417,550-570
Oppo et al., 2003	大西洋东北部	有孔虫	百年
Sicre et al., 2008	冰岛北部	冰芯	50-150
J. Zheng et al., 2010	中国东部、西部、青 藏高原	历史文献、树轮、 降水	200-300 百年
Perner et al., 2013	格陵兰西部	有孔虫	百年
Newby et al., 2014	北美洲	湖底沉积物	几百年
Thirumalai et al., 2018	Garrison 海盆	有孔虫	百年

石佳琪, 杨海军, 2021: 百年-千年气候变率: 观测、理论与模拟。科学通报, 待投稿。

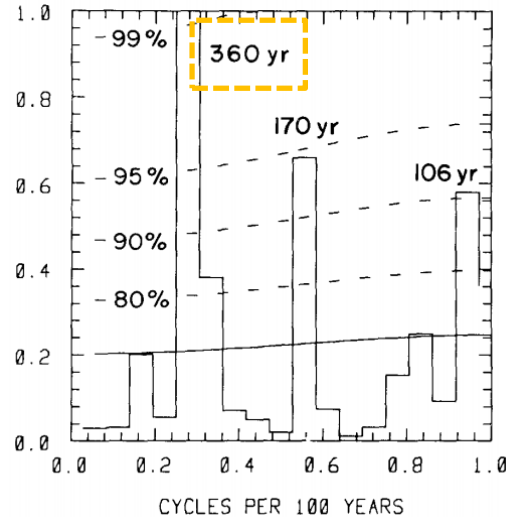


# Centennial Variability in *Proxy* Data

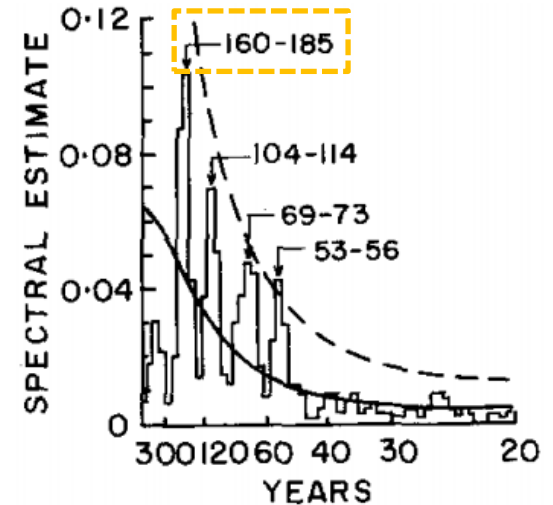
Lapland  
Tree rings width



Santa Barbara Basin sediment  
minimum population of hake



Camp Century  
Cores (氧同位素)

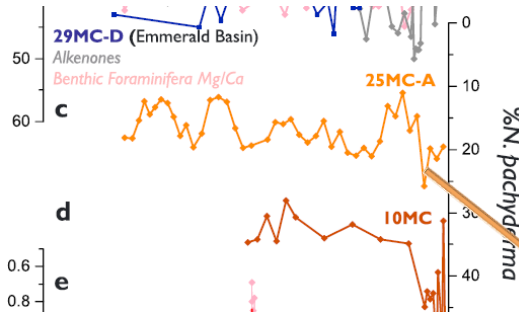
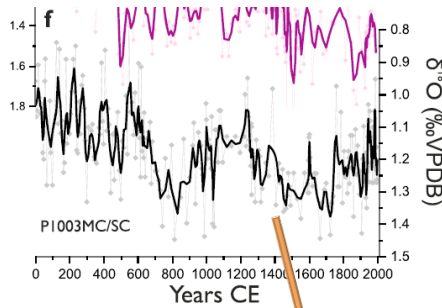


Siren et al. (1961), Lamb et al. (1977); Soutar and Issacs (1969); Johnsen and Dansgaard (1970)

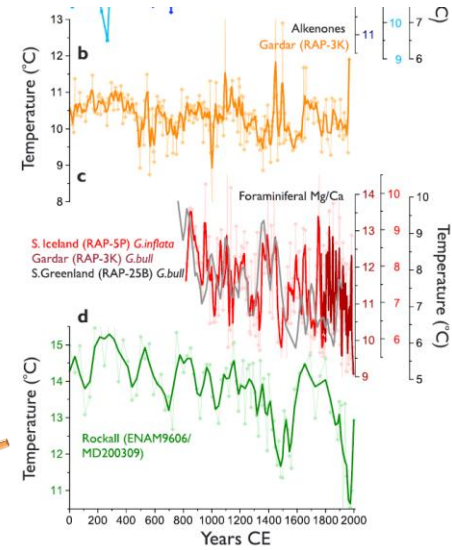
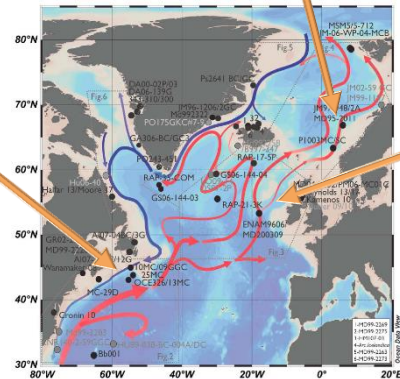


# Centennial Variability in *Proxy* Data

(f)  $\delta^{18}O_{foram}$  from P1003MC/SC



(c) % *N.pachyderma* from Laurentian Fan (25MC-A)

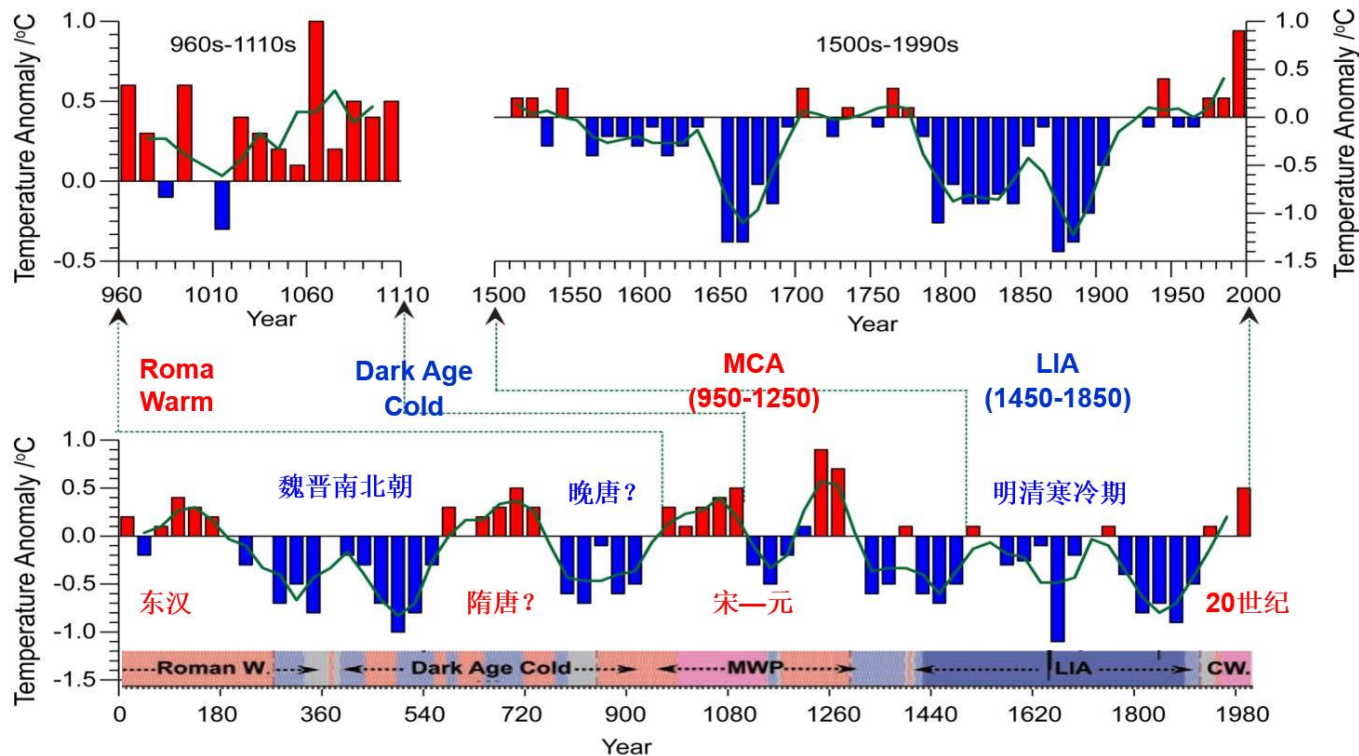


(b) alkenone records from RAPiD-21-3K;  
(d) foraminiferal Mg/Ca-based temperature reconstructions from Rockall Trough ENAM9606/M2003209

Moffa-Sánchez et al. (2019), Paleoceanography and Paleoclimatology



# Centennial Variability: 200-300 (?) Years



葛全胜, 郑景云, 满志敏, 方修琦, 张丕远, 2002: 过去2000a中国东部冬半年温度变化序列重建及初步分析. 地学前沿, 9(1), 169-181.





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1. Common Knowledge
2. Motivation
3. Observation
- 4. Theory, Simple Model: Previously**
5. Modeling

# 2-Box Model and Multi-Equilibrium

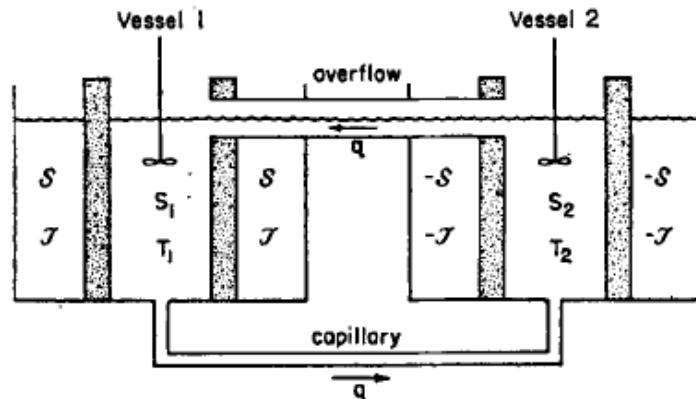
## Thermohaline Convection with Two Stable Regimes of Flow

By HENRY STOMMEL, Pierce Hall, Harvard University, Massachusetts

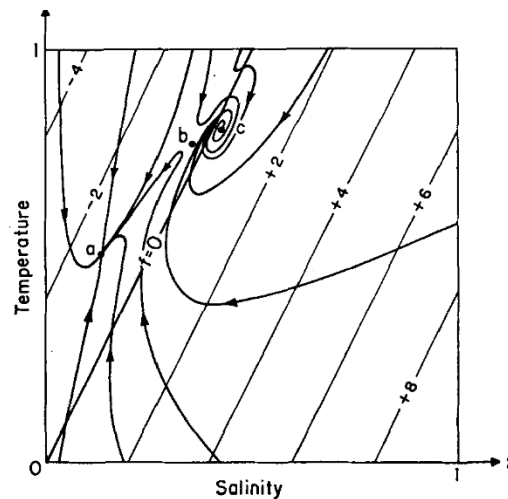
(Manuscript received January 21, 1961)

### Abstract

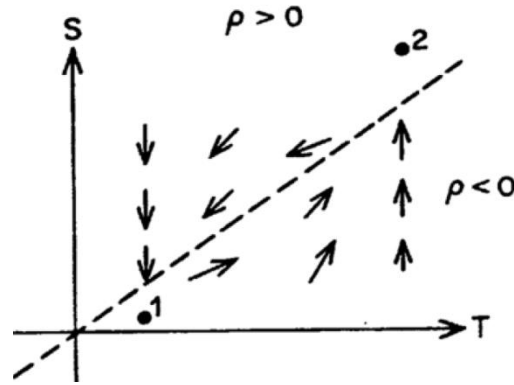
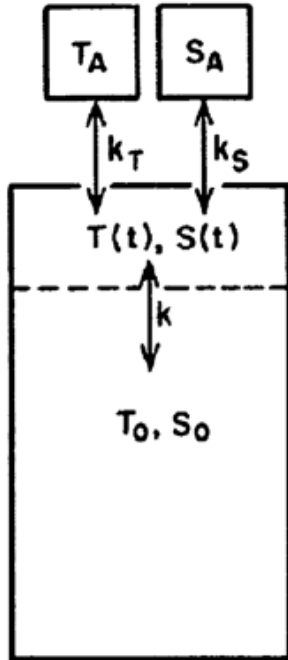
Free convection between two interconnected reservoirs, due to density differences maintained by heat and salt transfer to the reservoirs, is shown to occur sometimes in two different stable regimes, and may possibly be analogous to certain features of the oceanic circulation.



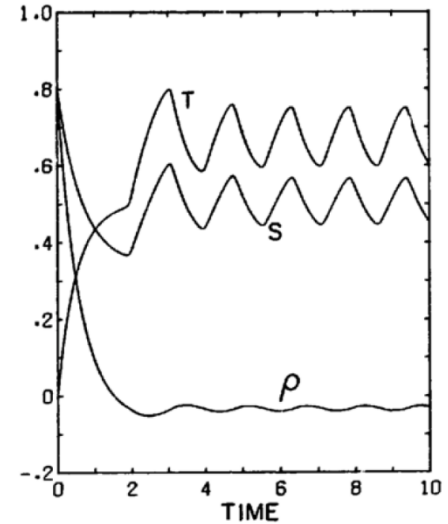
Henry Stommel (Tellus, 1961)



# Energy Source: *Ocean Convection*

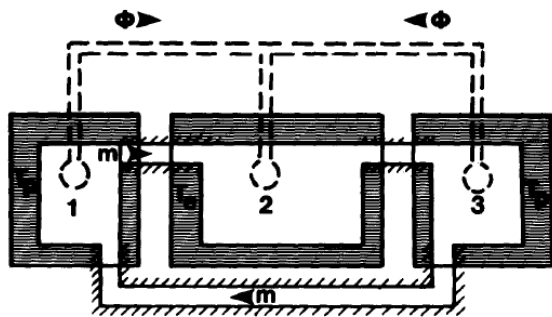


- *Flip-Flop model*
- **Self-sustained oscillation with increasing vertical turbulent mixing**

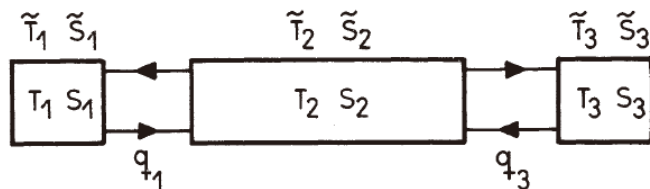


Pierre Welander (1982), A simple heat-salt oscillator. Dyn. Atmos. Oceans.

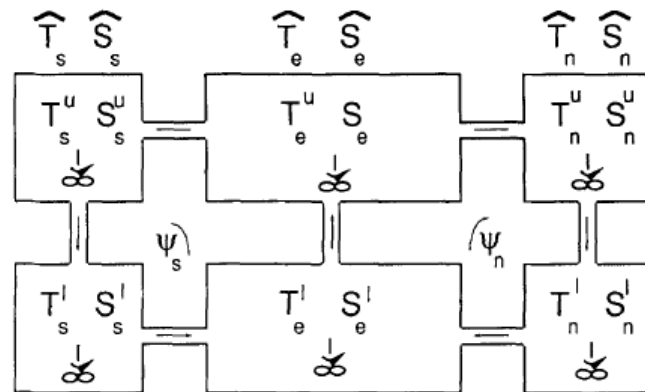
# 3-Box Model and Multi-Equilibrium



Claes Rooth (1982)



Pierre Welander (1986)

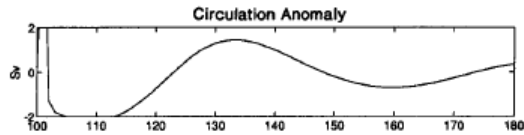


## 2D Model and 2-, 3-Box Model

Olivier Thual & James C.  
McWilliams (1992)

Climate transition between different stable regimes, with global and centennial-millennium timescale

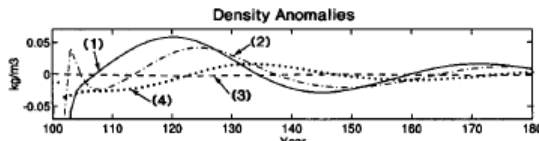
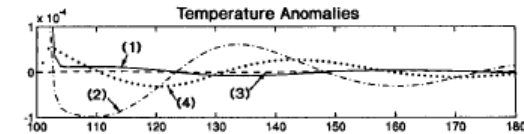
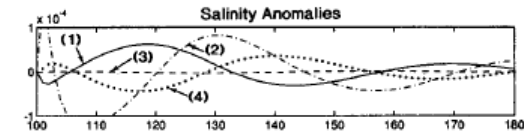
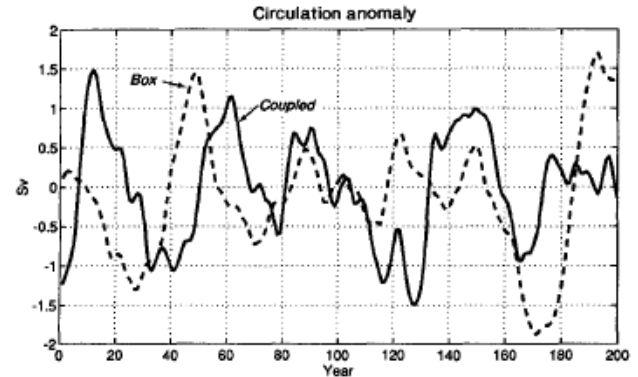
# Energy Source: *Atmosphere* Perturbation



←: Damped Oscillation

Mode

Circulation under random thermal forcing →



- 2-Box: Interdecadal variability of THC
- Linear interpretation
- Excited by atmospheric random forcing

Stephen Griffies and Eli Tziperman (1995): A linear thermohaline oscillator driven by stochastic atmospheric forcing. J. Climate

# Energy Source: *Ocean Advection* Feedback

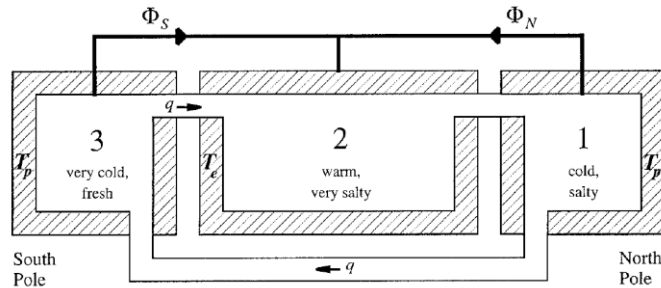
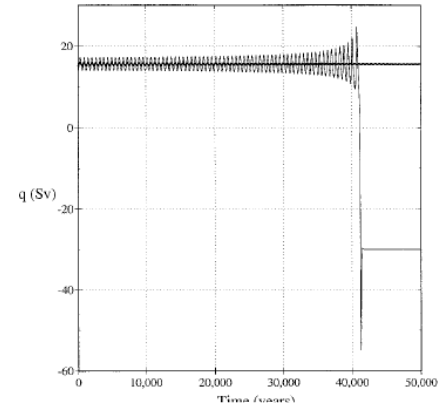
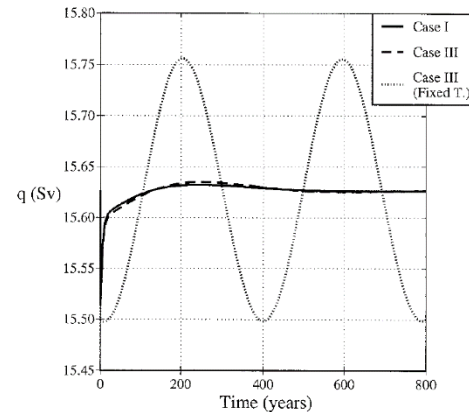


FIG. 1. Rooth's conceptual three-box model of thermohaline circulation, showing equilibrium conditions for Northern Hemisphere sinking. The separation between high- and low-latitude boxes is assumed to occur near the peak in atmospheric transports due to baroclinic eddy fluxes, i.e., about 35° latitude.



## 3-Box model with asymmetrical freshwater forcing / Stability of the equilibrium

### Periodic oscillation with constant Temperature / Collapse under some parameters

Jeffery Scott, Jochem Marotzke and Peter Stone (1999): Interhemispheric thermohaline circulation in a coupled box model. JPO.

# Single Equilibrium: *Self-Sustained Oscillation*

## Self-sustained oscillation with nonlinear close condition

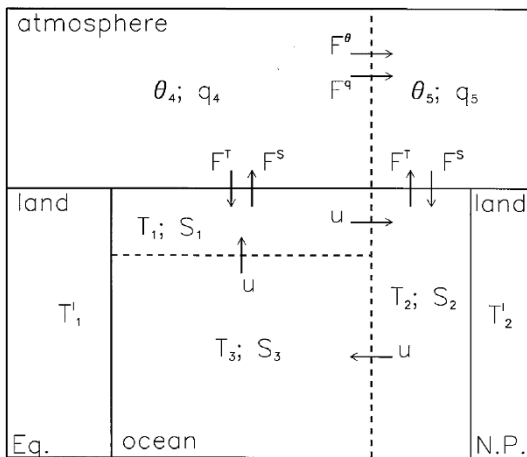
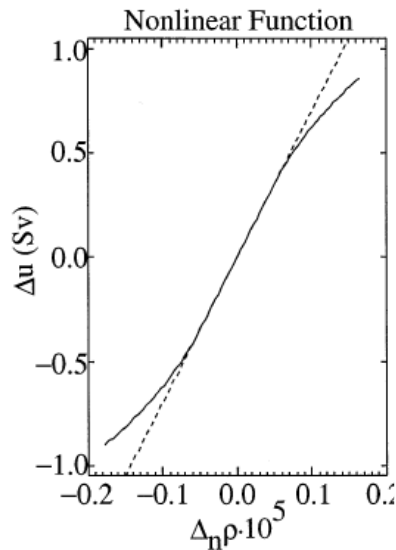


FIG. 1. The box model geometry.

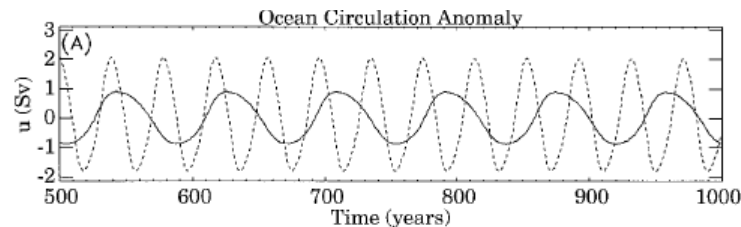
### 3-Box coupled model



$$u = \bar{u} + u' = \bar{u} + \xi(u_0, \Delta_n \rho') \Delta_n \rho', \quad (2)$$

where

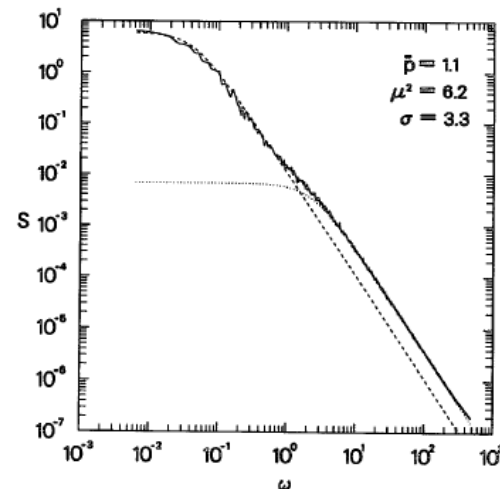
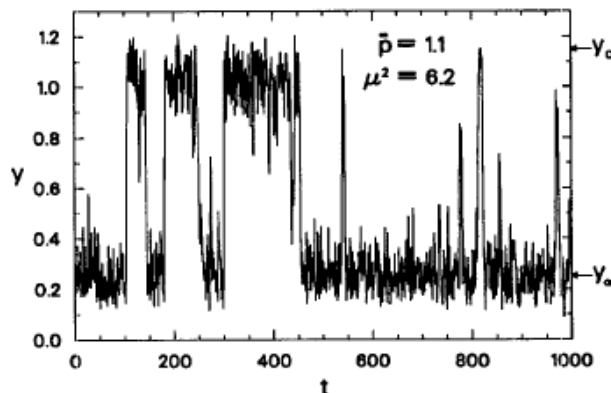
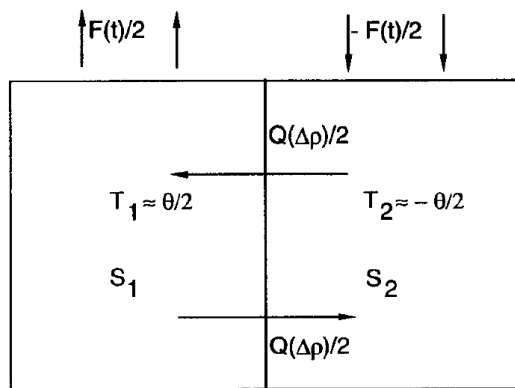
$$\xi(C, x) = \begin{cases} C \frac{x_+}{x} \left[ k \left( \frac{x}{x_+} \right)^{1/k} - 1 \right] + 1 & \text{if } x > x_+ \\ C & \text{if } x_+ \geq x \geq x_- \\ C \frac{x_-}{x} \left[ k \left( \frac{x}{x_-} \right)^{1/k} - 1 \right] + 1 & \text{if } x < x_- \end{cases} \quad (3)$$



Rivlin & Tziperman (1997): Linear versus self-sustained interdecadal thermohaline variability in a coupled box model. JPO

# Multi-Equilibrium: *Forced Regime Shift*

Stommel 2-Box model, no *intrinsic* variability, stochastic forced variability



**Middle: Multi-equilibrium and forced oscillation; Right: Power spectrum**

Paola Cessi (1994), A simple box model of stochastically forced thermohaline flow. JPO



# Single Equilibrium: *Forced Oscillation*

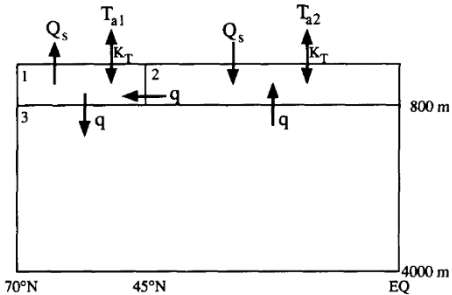
## Atmosphere Lorenz model and Ocean 3-Box model

Lorenz (1984, 1990) introduced a low-order atmospheric "general circulation" model, defined by three ordinary differential equations:

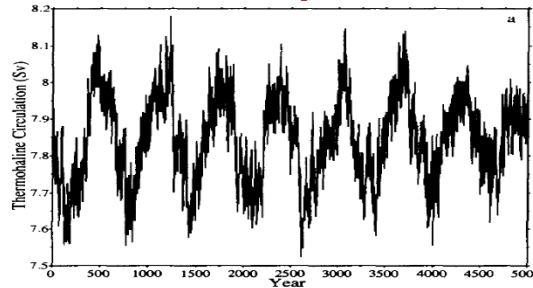
$$\frac{dX}{dt} = -Y^2 - Z^2 - aX + aF, \quad (1)$$

$$\frac{dY}{dt} = XY - bXZ - Y + G, \quad (2)$$

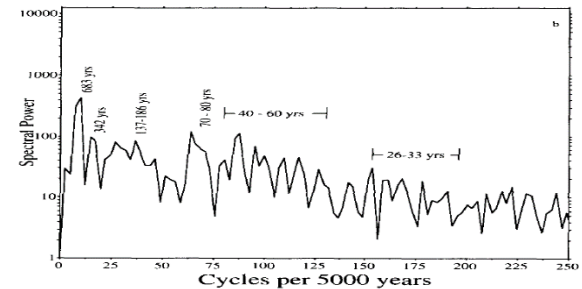
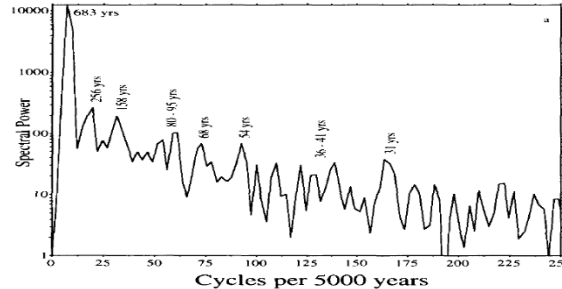
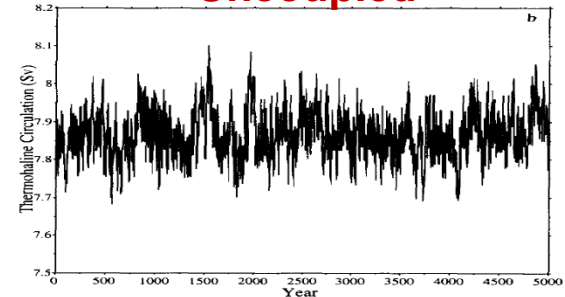
$$\frac{dZ}{dt} = bXY + XZ - Z. \quad (3)$$



### Coupled



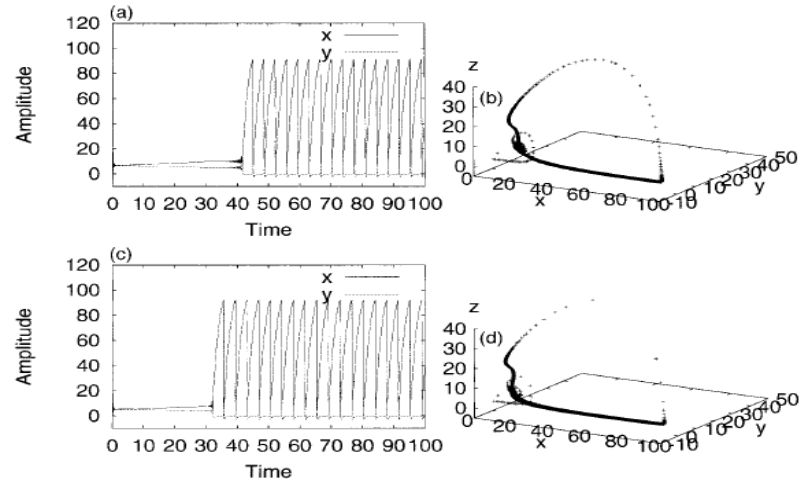
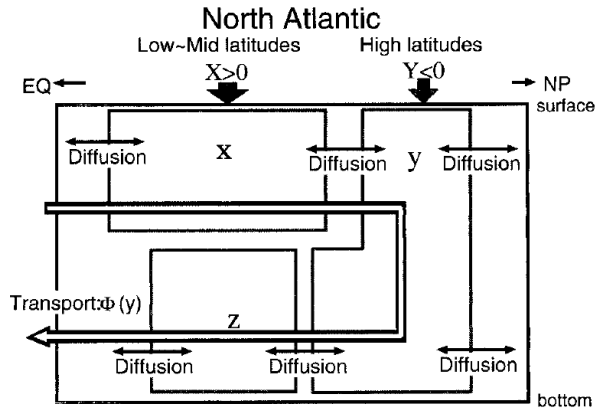
### Uncoupled



Paul Roebber (1995), Climate variability in a low-order coupled atmosphere-ocean model. Tellus-A

# 3-Box Model for Bond Cycle

A 3-Box with only Salinity considered, internal *Millennial* oscillation



**Bifurcation: from a stable solution to an unsteady bounded oscillation**

Sakai & Peltier (1999), A dynamical systems model of the Dansgaard-Oeschger oscillation and the origin of the bond cycle. JC

# Thermohaline Circulation Stability: *Regime Shift*

## 3-Box model, hysteresis behavior under freshwater forcing

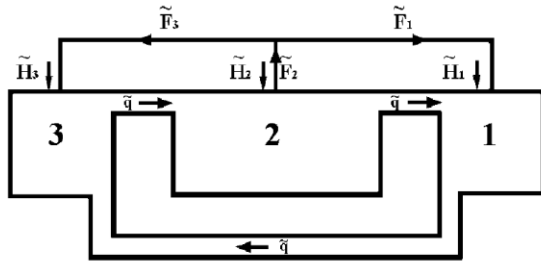
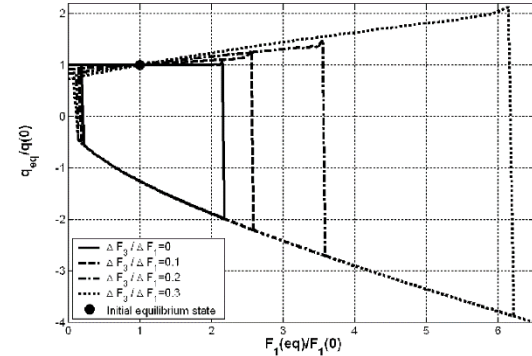
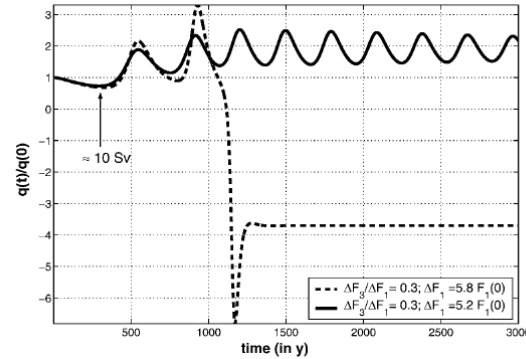


FIG. 1. Schematic picture of the interhemispheric box model.



Lucarini & Stone (2005), Thermohaline circulation stability: a box model study. Part I: uncoupled model. JC

# Thermohaline Circulation Centennial Oscillation

2-D with random forcing, **200-300** years oscillation

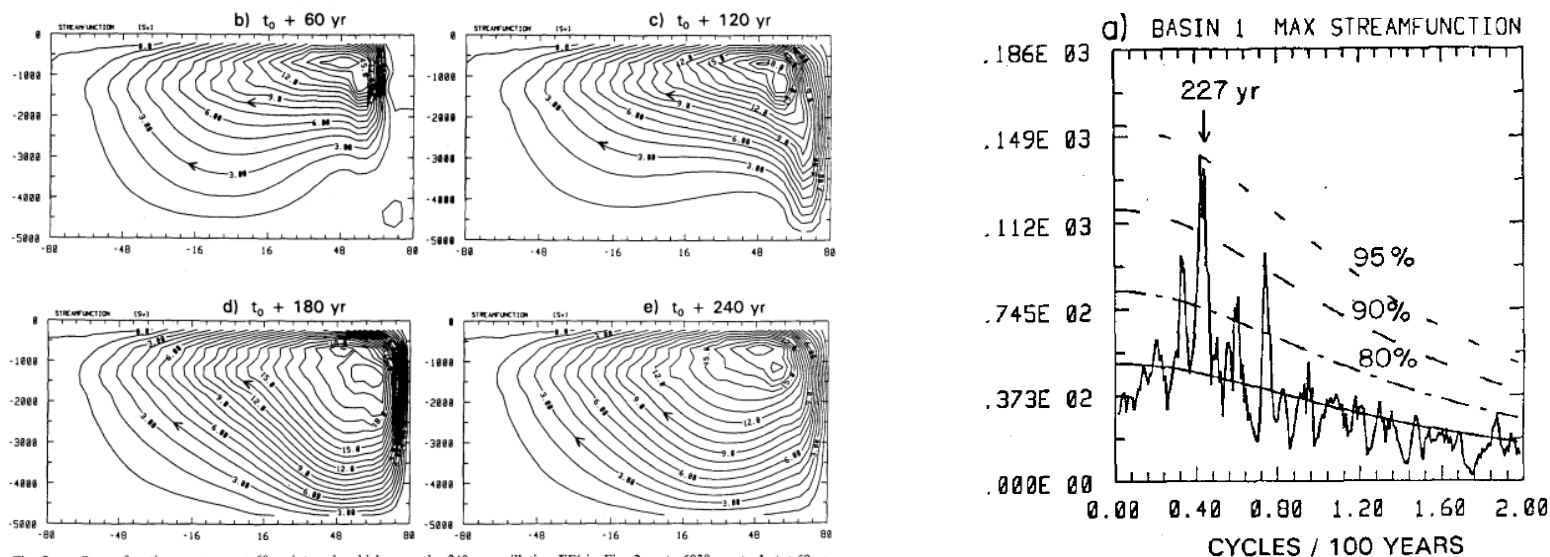


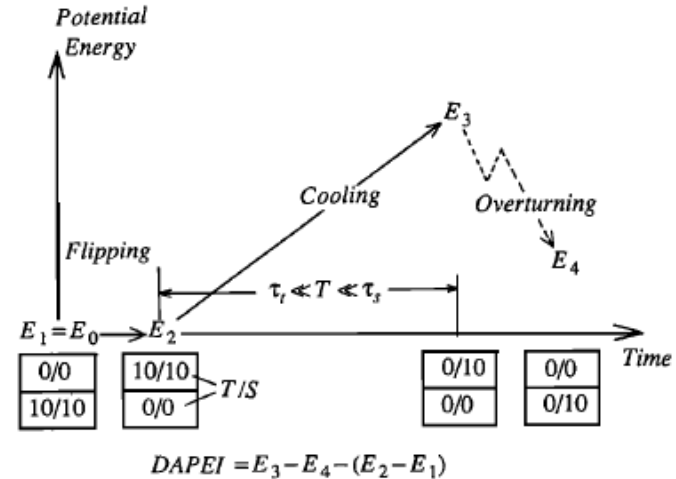
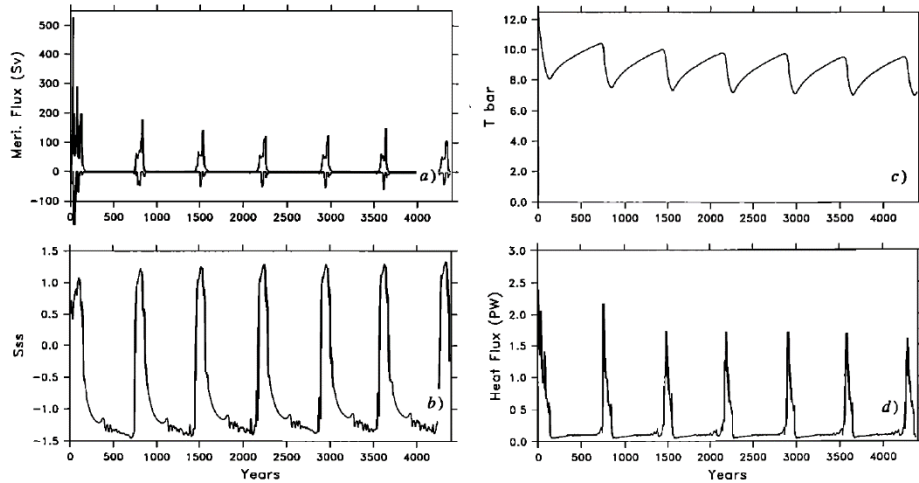
Fig. 3a-e. Streamfunction contours at 60-yr intervals which span the 240-yr oscillation EE' in Fig. 2. a  $t = 6830 \text{ yr} = t_0$ ; b  $t_0 + 60$  yr; c  $t_0 + 120$  yr; d  $t_0 + 180$  yr, and e  $t_0 + 240$  yr (end of oscillation)

**Period: 200-300 years of AMOC, Salinity advection feedback**

Mysak et al., Climate Dynamics, 1993: Century-scale variability in a randomly forced, 2-D thermohaline ocean circulation model.

# Energy Source: *Ocean Convection*

## 3-D OGCM with freshwater forcing, *centennial-millennial* oscillation

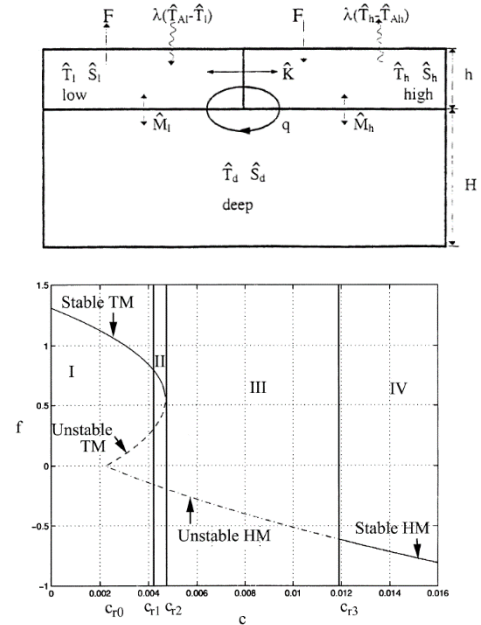
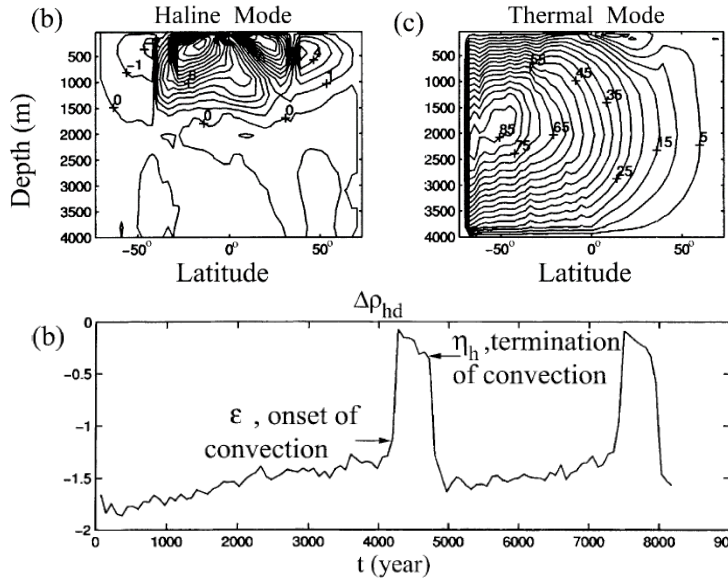
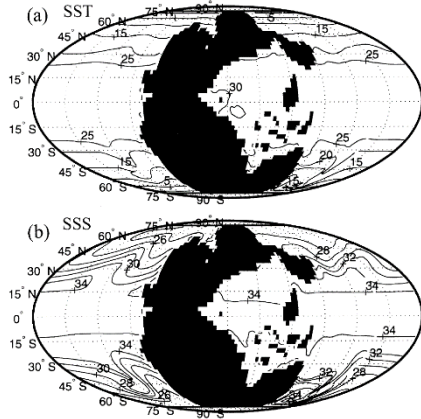


- Periodic oscillation of saline mode
- DAPE provides energy to saline mode's oscillation

Huang (1994): Thermohaline circulation: Energetics and variability in a single-hemisphere basin model. JGR-ocean

# Multi-Equilibrium: *Self-Sustained Oscillation*

## Late Permian, Equable climate and regime shift, *Millennial* oscillation

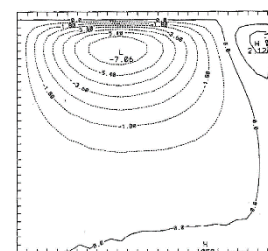
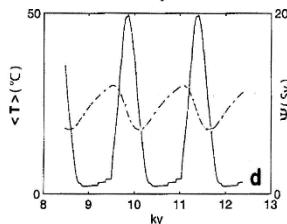
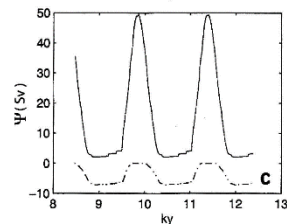
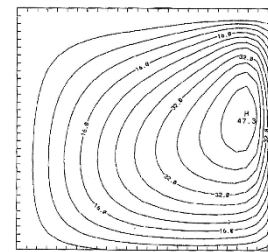
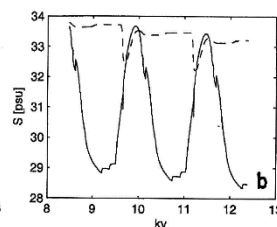
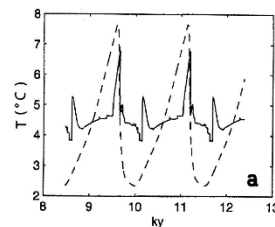
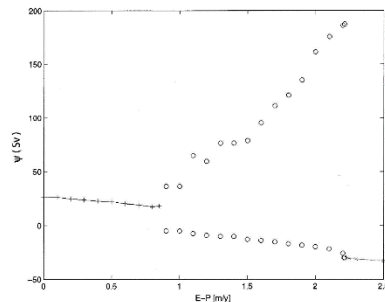
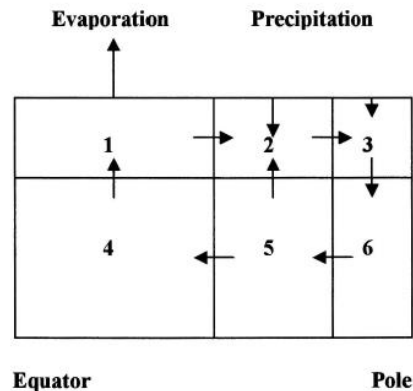


Such oscillations do not appear to occur in the modern ocean, because, apparently, the surface freshwater forcing is not strong enough. Mode switching is more likely to occur, perhaps, during glacial periods in which the freshwater forcing due to ice melting at polar regions is much stronger, or during warm equable paleoclimates such as the late Permian, or mid-Cretaceous in which the buoyancy forcing due to freshwater flux may have been stronger than the air-sea heat flux

Zhang et al. (2002), Mechanism of thermohaline mode switching with application to warm equable climates. *JC*

# Multi-Equilibrium: *Self-Sustained Oscillation*

Lowest-order 3x2-Box and 2D model, internal *Millennial* oscillation

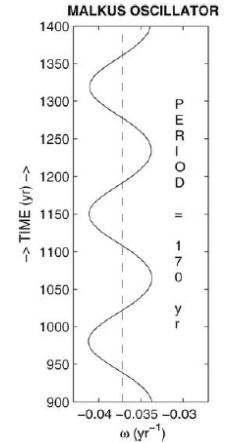
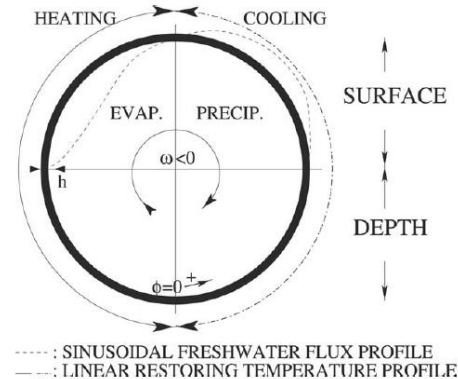
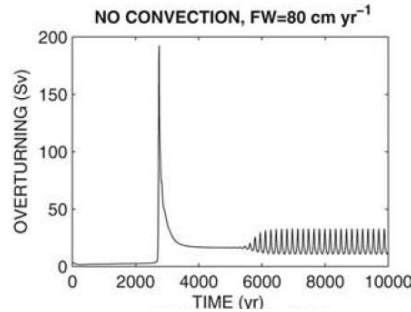
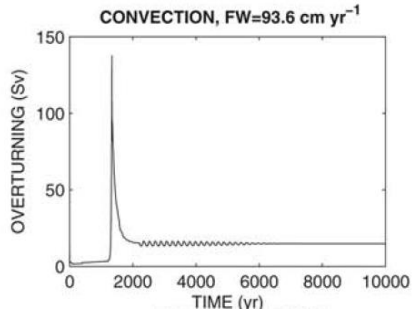


E-P increasing: stable **thermal mode** → **oscillation mode** → steady **haline mode**.

Colin De Verdière, Jelloul and Sevellec (2006), Bifurcation structure of thermohaline millennial oscillations. JC

# Beyond Box Model

## 2-D model and 1-D Howard-Malkus loop model, internal *Centennial* oscillation



Left: 2-D model; Right: 1-D model of Howard-Malkus loop

Not self-sustained: either strong damped or runaway mode

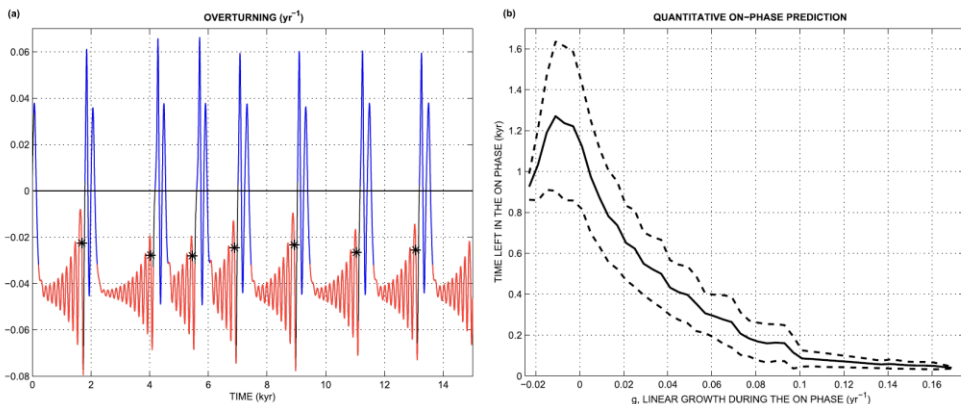
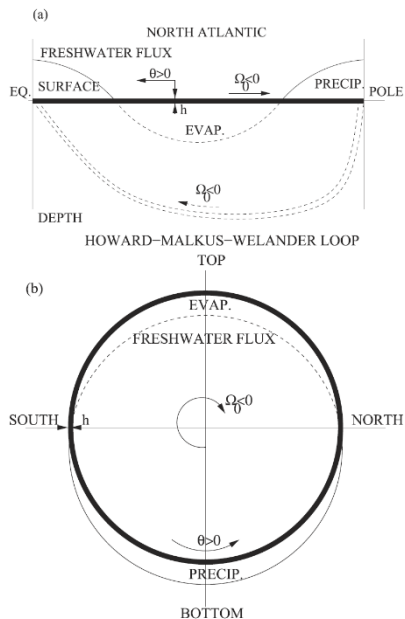
Nonlinear or linear; convection or no convection

Sévellec et al. (2006), On the mechanism of centennial thermohaline oscillations. J. Marine Research



# Beyond Box Model

## 1-D Howard-Malkus loop model, AMOC *Millennial* regime shifts



**AMOC *Millennial* shift is predictable in this chaotic model**  
**Two predictive indices are defined**

Sévellec & Fedorov (2014), Millennial variability in an idealized model: predicting the AMOC regime shifts

# Centennial Oscillation in Coupled GCM

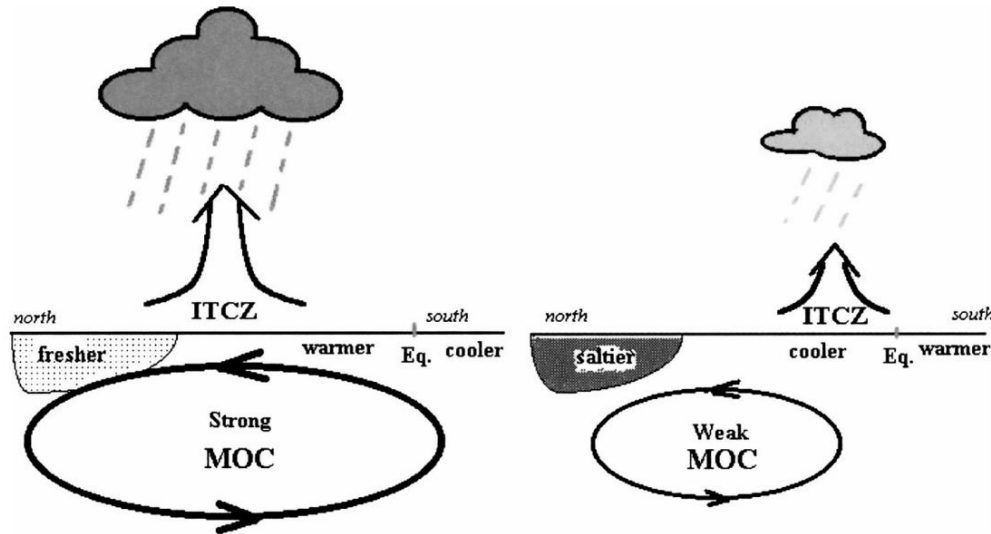


FIG. 16. Schematic of mechanism responsible for centennial THC fluctuation in HadCM3. When the THC is (left) strong ITCZ shifts northward, in response to enhanced SST gradient across equator. Fresh anomalies in the upper-ocean propagate northward and weaken the overturning. This results in the (right) weak phase.

**AMOC**  $\uparrow$   $\rightarrow$

$\rightarrow$  **Northward OHT**  $\uparrow$

$\rightarrow$  **Cross Eq.  $\Delta$ SST**  $\uparrow$

$\rightarrow$  **ITCZ Northward Rain**  $\uparrow$

$\rightarrow$  **Tropical Salinity**  $\downarrow$

$\rightarrow$  **Northward S-advection**  $\downarrow$

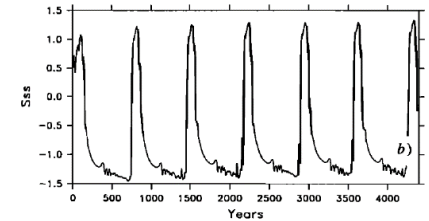
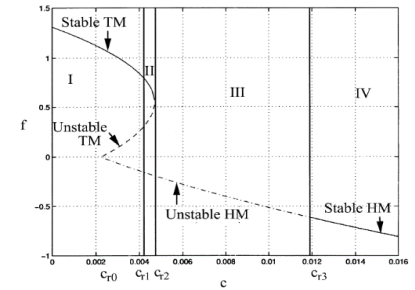
$\rightarrow$  **NADW Salinity**  $\downarrow$

$\rightarrow$  **AMOC**  $\downarrow$

Vellinga and Wu (2004), Low-latitude freshwater influence on centennial variability of the Atlantic THC. JC

# Previous Theoretical Studies: *Summary*

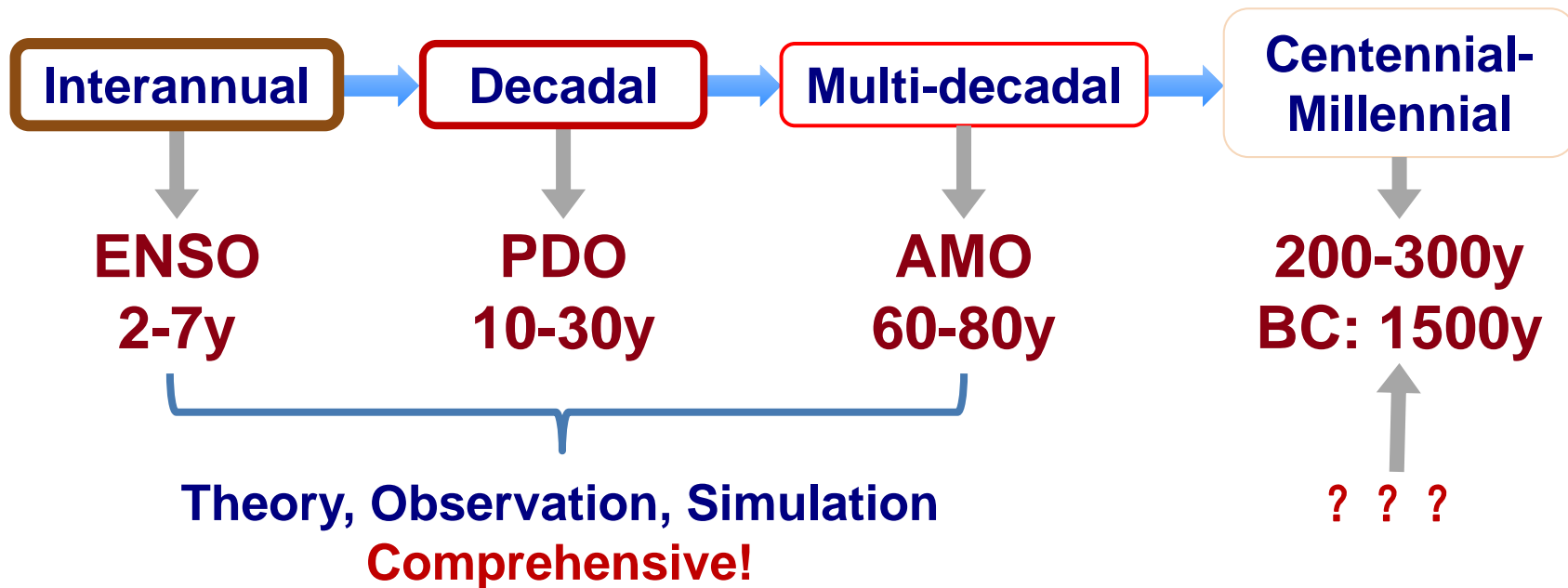
- **THC: stability, bifurcation and regime shift**
- **Forcing: freshwater or/and stochastic**
- **Transition: thermal mode to haline mode**
- **Self-sustained oscillation:  $\delta$ -function-like**
- **Not particularly on Holocene**



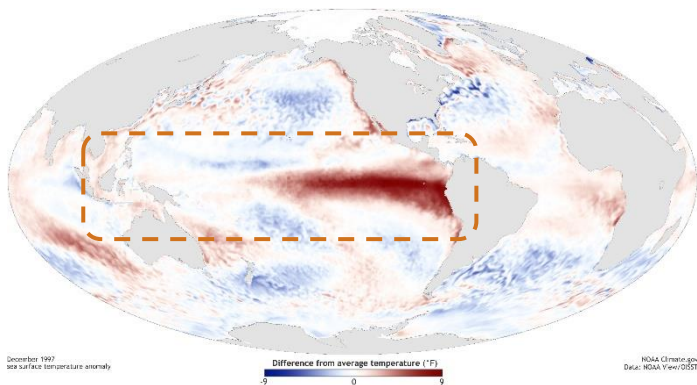
# Contents

1. Common Knowledge
2. Motivation
3. Observation
- 4. Theory: Our Paradigm**
5. Modeling

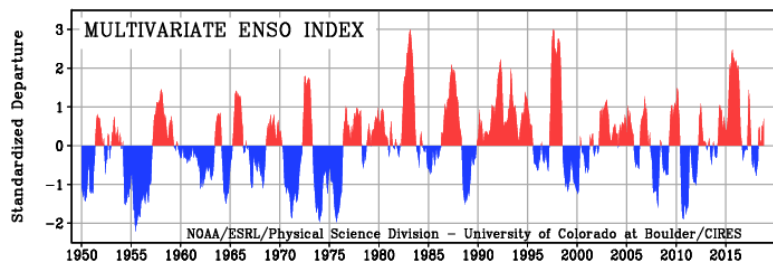
# Climate Variability that *Ocean Matters*



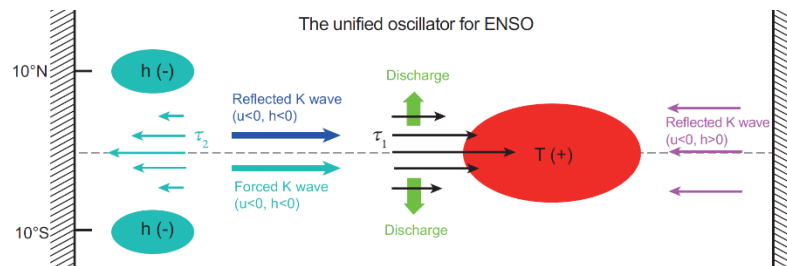
# ENSO Variability: 2-7 Years



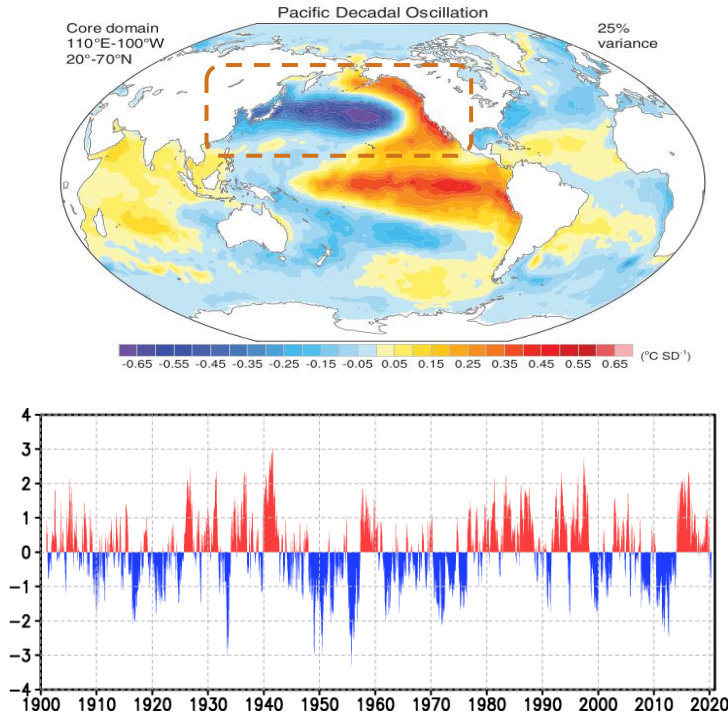
- Eigen Mode : **E**quatorial **B**asin **M**ode, Rossby + Kelvin waves. Cane and Moore (1981)
- Mixed Layer + Thermocline (200m)
- Wang et al. (2018), NSR: A Review of ENSO theories



<https://www.psl.noaa.gov/ens/mei.old/>



# Decadal Variability (PDO): 10-30 Years

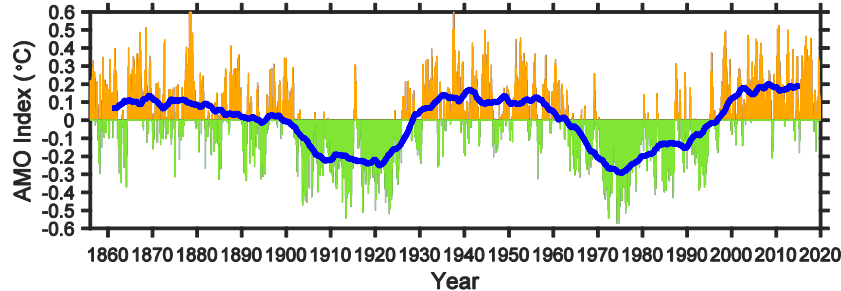
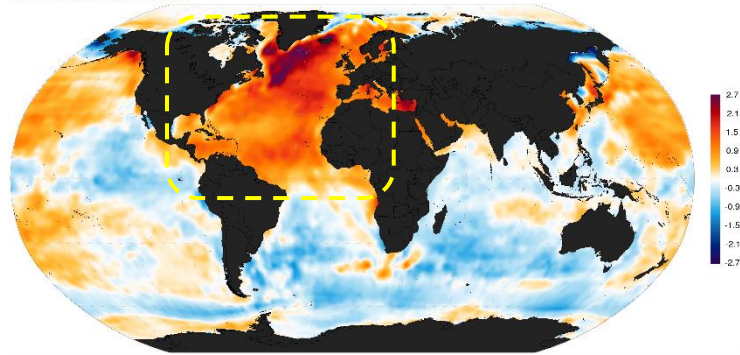


- Eigen Mode:
  1. Planetary Wave Basin Mode, Extratropical Rossby + Coastal Kelvin waves. Yang and Liu (2003)
  2. Subduction mode, 2<sup>nd</sup> Rossby wave. Gu and Philander (1997); Liu (1999a, b, 2003)
- Mixed Layer + Thermocline (500m)
- Liu (2012), JC: Dynamics of Interdecadal Climate Variability: A Historical Perspective
- Liu and Di Lorenzo (2018), Current Climate Change Reports: Mechanisms and Predictability of Pacific Decadal Variability

[https://www.daculaweather.com/4\\_pdo\\_index.php](https://www.daculaweather.com/4_pdo_index.php)

# Multi-Decadal Variability (AMO): 60-80 Years

Atlantic Multidecadal Oscillation

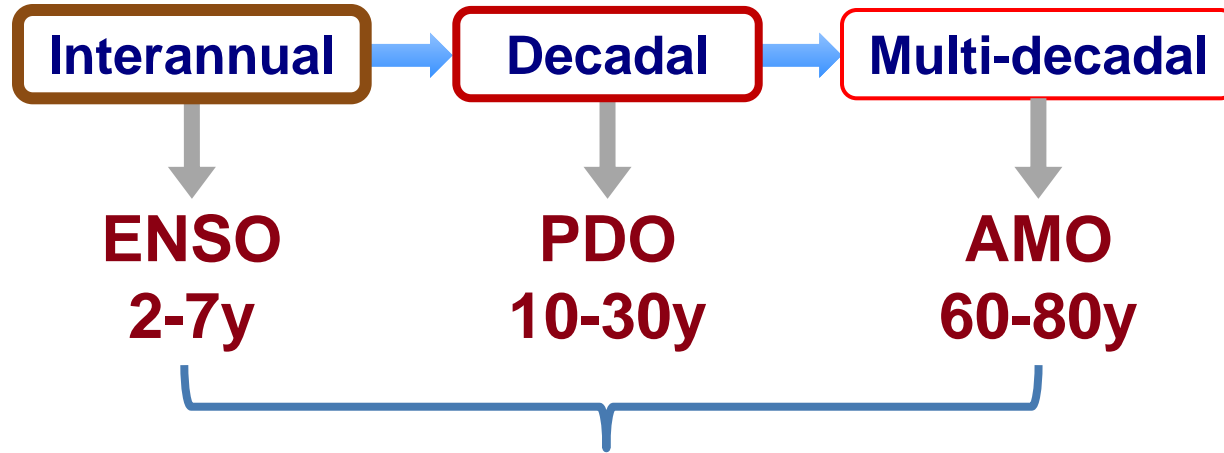


- Eigen Mode:
  1. Planetary Wave Basin Mode, Extratropical Rossby Waves
  2. Advection mode of AMOC
- Thermohaline dynamics (4000m)
- Vecchi, Delworth and Booth (2018), Nature: Origins of Atlantic Decadal Swings
- Drinkwater et al. (2014), JMS: The Atlantic Multidecadal Oscillation: Its manifestations and impacts with special emphasis on the Atlantic region north of 60N

[https://en.wikipedia.org/wiki/Atlantic\\_multidecadal\\_oscillation](https://en.wikipedia.org/wiki/Atlantic_multidecadal_oscillation); UK Met Office; HadISST  
<https://climatedataguide.ucar.edu/climate-data/atlantic-multi-decadal-oscillation-amo>



# Climate Variability that *Ocean Matters*



Theory: Eigen mode excited by  
**Air-sea coupling**  
**External (Random) forcing**

We would like to

Searching for **Eigen Mode**

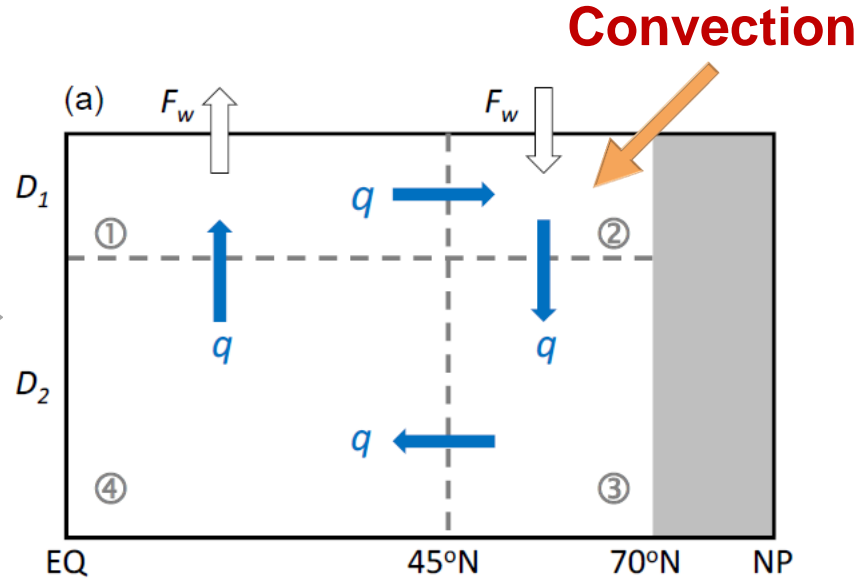


**Centennial-Millennial Timescale Climate Variability**



? ? ?

# One Hemisphere 4-Box Model



Li and Yang (2021)



# Convection

Static instability (头重脚轻)

Convective instability (头冷脚热)



Adiabatic APE

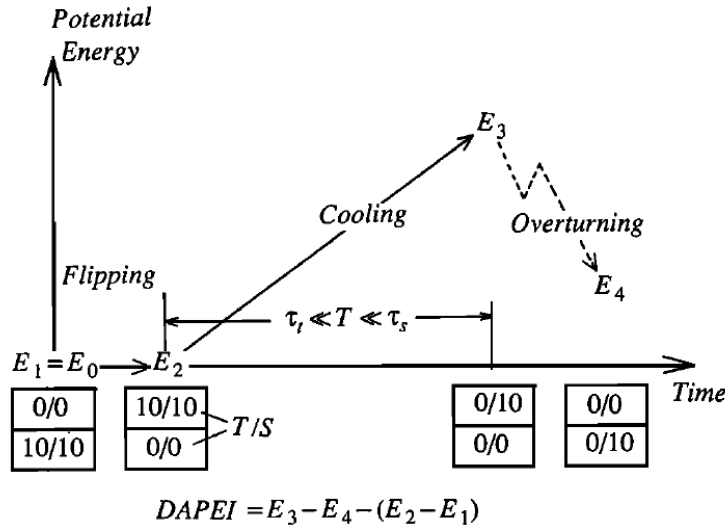


Diabatic APE

Li and Yang (2021)

# Convection Instability

## Diabatic Available Potential Energy (DAPE)



Huang (1994)

During the transition from state 3 to 4, a large amount of potential energy is released. The diabatic available potential energy index (DAPEI) is defined as the energy released during the overturning minus the energy required to push the slightly heavy water from the lower box upward to initiate the whole process,

$$DAPEI = E_4 - E_3 - (E_2 - E_1) = \rho_0 g h^2 \alpha (T_2 - T_1)$$

$$DAPEI = \iiint_0^H g \rho_0 \alpha [T(z) - T_{\text{surf}}] (H - z) \delta \, dz \, dx \, dy$$

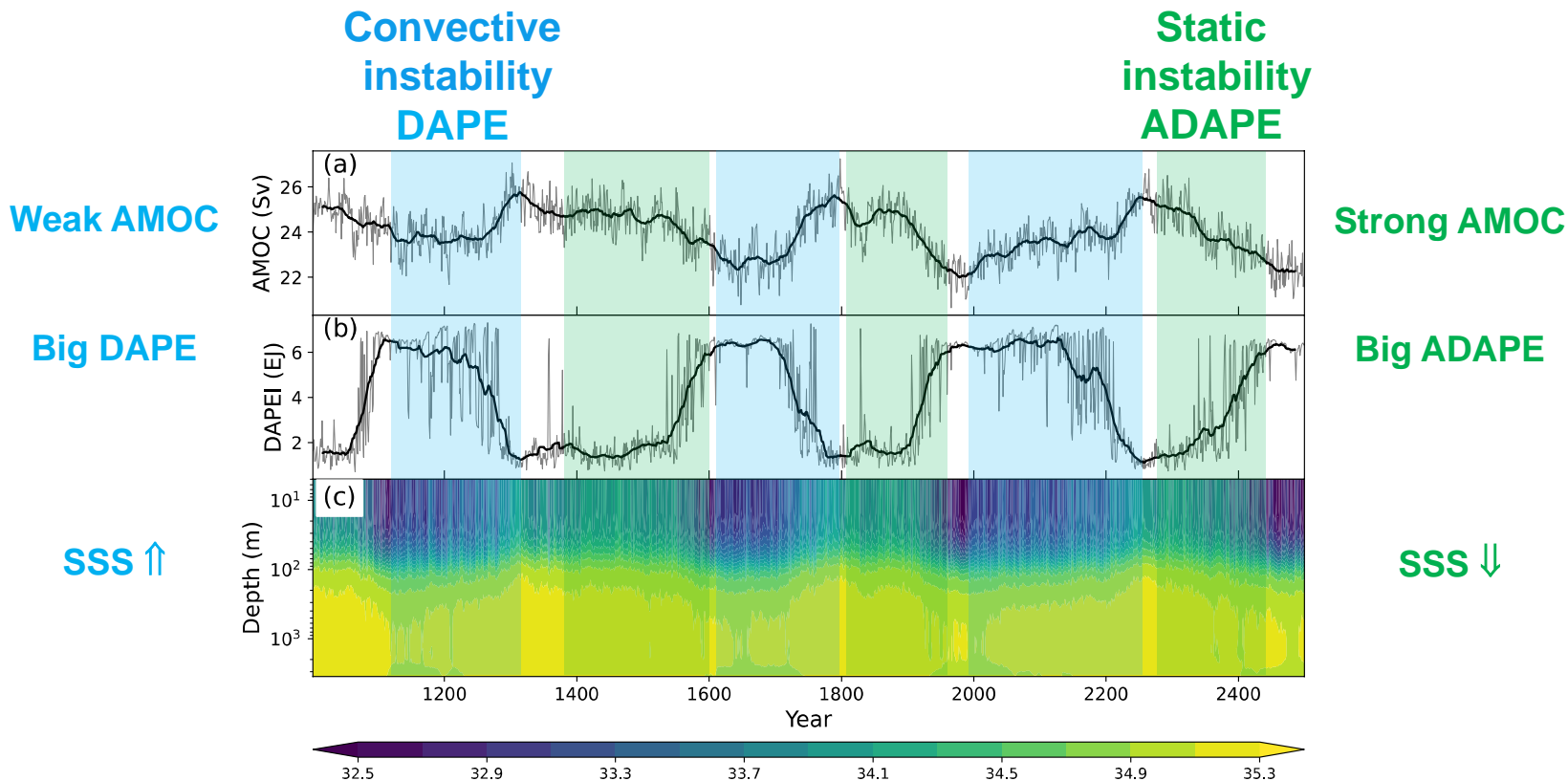
where  $\delta$  is a switch,  $\delta = 1$  if  $T(z) > T_s$  and  $S(z) > S_s$ , and  $\delta = 0$  otherwise. Thus the switch  $\delta$  is on only if the stratification is in favor of convective overturning.

In comparison, the classic APE is defined as

$$APE = \iiint -\frac{g \rho'^2}{2} \left( \frac{\partial \bar{\rho}}{\partial z} \right)^{-1} \, dx \, dy \, dz,$$

where  $\partial \bar{\rho} / \partial z$  is the horizontal mean of the vertical density gradient.

# AMOC, DAPE and Upper Salinity in CESM

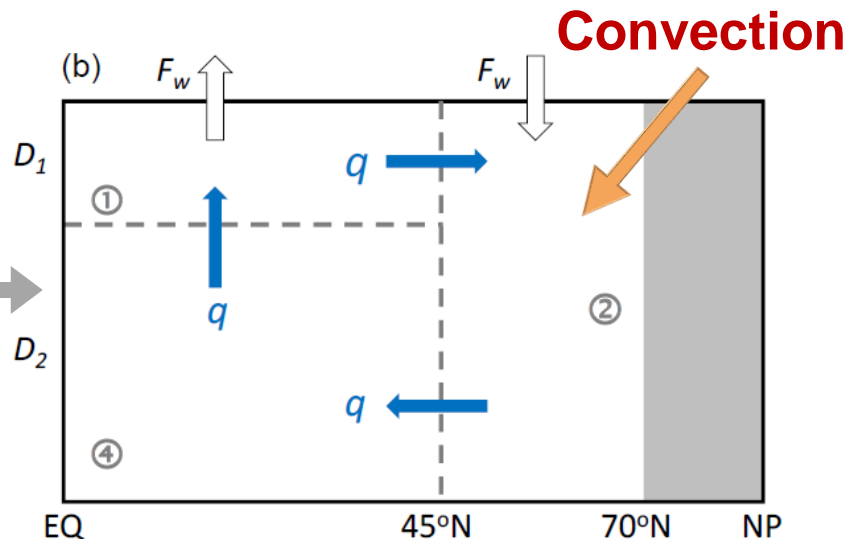
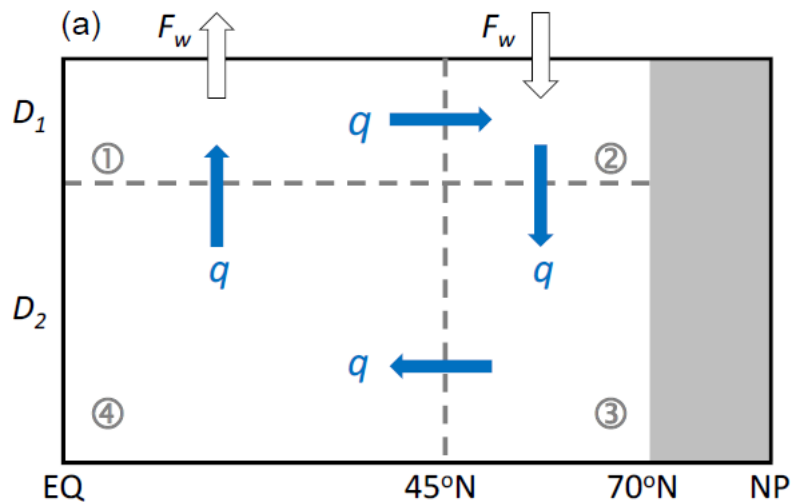


Li and Yang (2021)



# One Hemisphere Box Model

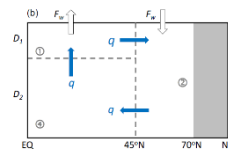
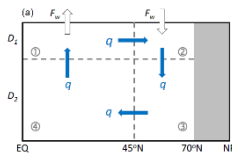
$$|S_2 - S_3| > S_b \Rightarrow \begin{cases} S_2 - S_3 > S_b^+ , & \text{Static instability} \\ S_2 - S_3 < S_b^- , & \text{Convective instability} \end{cases}$$



Li and Yang (2021)



# One Hemisphere Box Model



$$V_1 \dot{S}'_1 = q'(\bar{S}_4 - \bar{S}_1) + \bar{q}(S'_4 - S'_1)$$

$$V_2 \dot{S}'_2 = q'(\bar{S}_1 - \bar{S}_2) + \bar{q}(S'_1 - S'_2)$$

$$V_3 \dot{S}'_3 = q'(\bar{S}_2 - \bar{S}_3) + \bar{q}(S'_2 - S'_3)$$

$$V_4 \dot{S}'_4 = q'(\bar{S}_3 - \bar{S}_4) + \bar{q}(S'_3 - S'_4)$$



$$V_1 \dot{S}'_1 = q'(\bar{S}_4 - \bar{S}_1) + \bar{q}(S'_4 - S'_1)$$

$$V_2 \dot{S}'_2 = q'(\bar{S}_1 - \bar{S}_2) + \bar{q}(S'_1 - S'_2)$$

$$V_4 \dot{S}'_4 = q'(\bar{S}_2 - \bar{S}_4) + \bar{q}(S'_2 - S'_4)$$

$$V_1 S'_1 + V_2 S'_2 + V_4 S'_4 = \text{constant}$$

$$\Delta\rho' = \rho_0\beta[\delta(S'_2 - S'_1) + (1 - \delta)(S'_3 - S'_4)], \text{ and } \delta = \frac{V_1}{V_1 + V_4} = \frac{V_2}{V_2 + V_3} = \frac{D_1}{D}$$

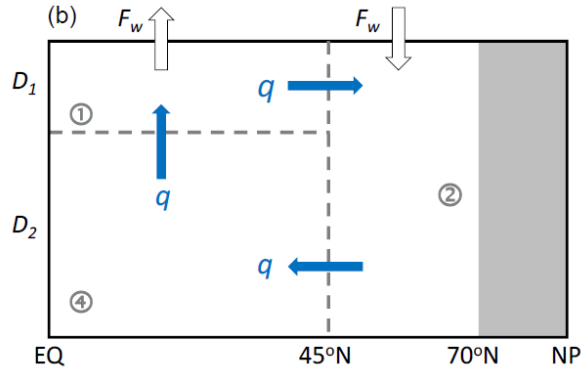
$$q' = \lambda\Delta\rho' = \lambda\rho_0\beta[S'_2 - \delta S'_1 - (1 - \delta)S'_4], \text{ and } \delta = \frac{V_1}{V_1 + V_4} = \frac{D_1}{D}$$

Li and Yang (2021)





# Theoretical Solution to 3-Box Model



Li and Yang (2021)

$$\omega = \frac{1}{2} \left[ (C_2 M - C_3) \pm \sqrt{(C_2 M - C_3)^2 - 4 C_2 C_4 (1 - M)} \right]$$

where  $C_1 = \frac{1}{\delta_1} + \frac{1}{\delta_2}$ ,  $C_2 = \frac{1}{\delta_1 \delta_2}$ ,  $C_3 = \frac{1}{\delta_1} + \frac{1}{\delta_2} + \frac{1}{\delta_4}$ , and  $C_4 = \frac{1}{\delta_4}$ .

Based on (A5), the essential stability condition for the system is

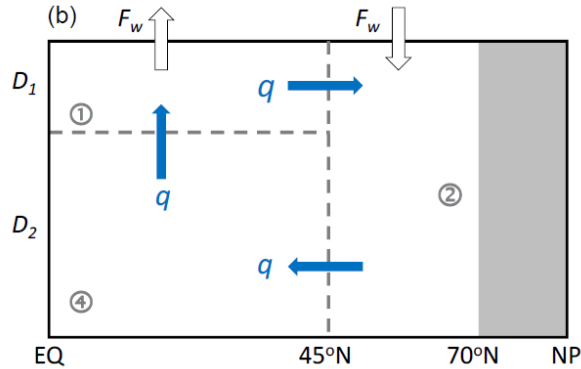
$$M \leq \min\left(\frac{C_3}{C_2}, 1\right)$$

and the oscillation condition of the system is

$$M_1 < M < \min(M_2, 1)$$

where  $M_{1,2} = \frac{C_3 - 2C_4}{C_2} \pm \frac{2}{C_2} \sqrt{C_4^2 + C_4(C_2 - C_3)}$ . Thus,  $\lambda_{1,2} = \bar{q} M_{1,2} / \rho_n$ .

# Stability Condition for 3-Box Model

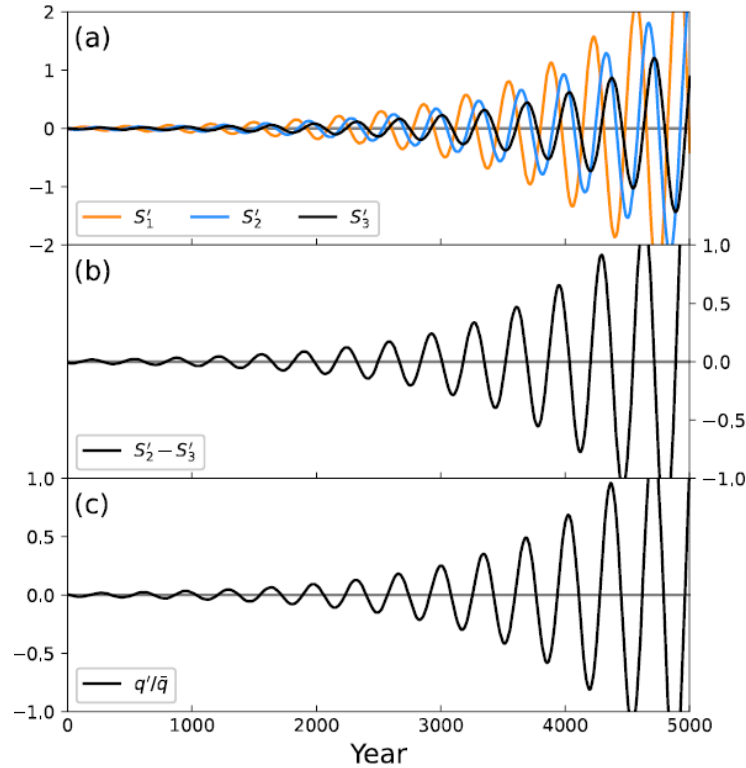


$$\lambda \leq \lambda_C \equiv \frac{\bar{q}^2}{\rho_0 \beta F_w} \frac{(V_1 + V_4)(V_1 V_2 + V_2 V_4 + V_1 V_4)}{V_1 V_4 (V_1 + V_2 + V_4)}$$

Here,  $\lambda_C$  is defined as the critical value to the linear closure parameter  $\lambda$ , which is determined by mean THC strength  $\bar{q}$ , the atmosphere moisture flux  $F_w$  as well as the basin geometry  $V_i$ . This parameter implies the complexity of oscillation behaviors in reality: at the multi-centennial timescale, whether an oscillation mode can be identified from paleoclimatic proxy data is full of uncertainty.

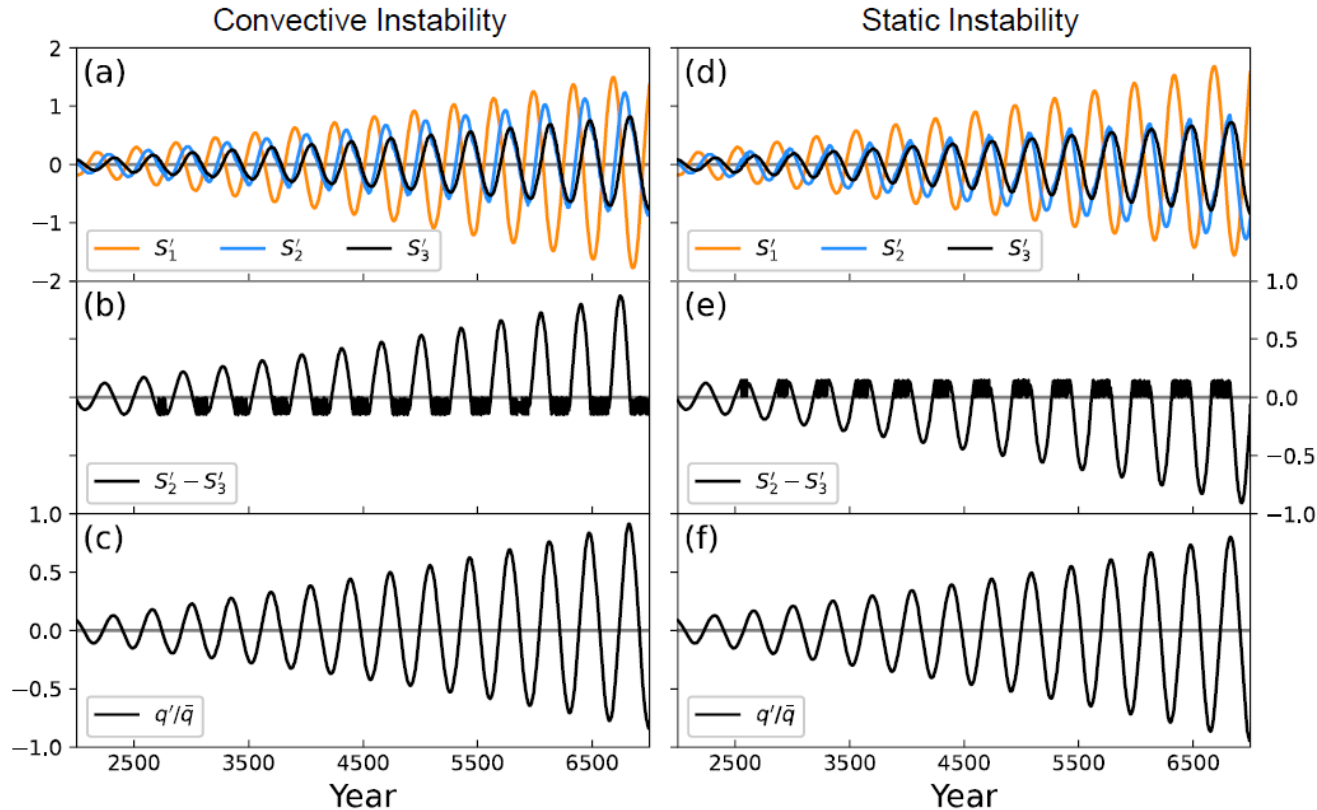
Li and Yang (2021)

# Unstable Oscillation without Convection



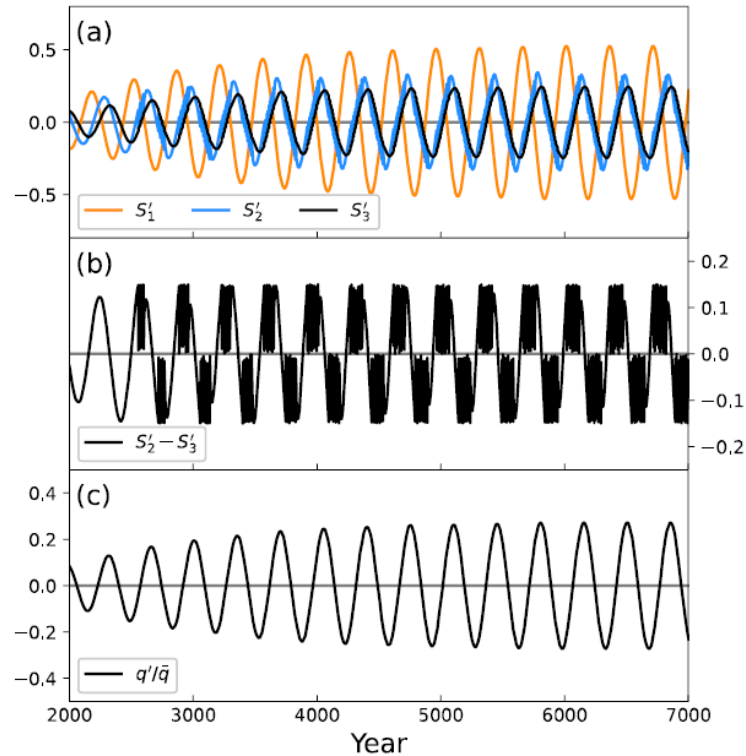
Li and Yang (2021)

# Unstable Oscillation with *One* Convection



Li and Yang (2021)

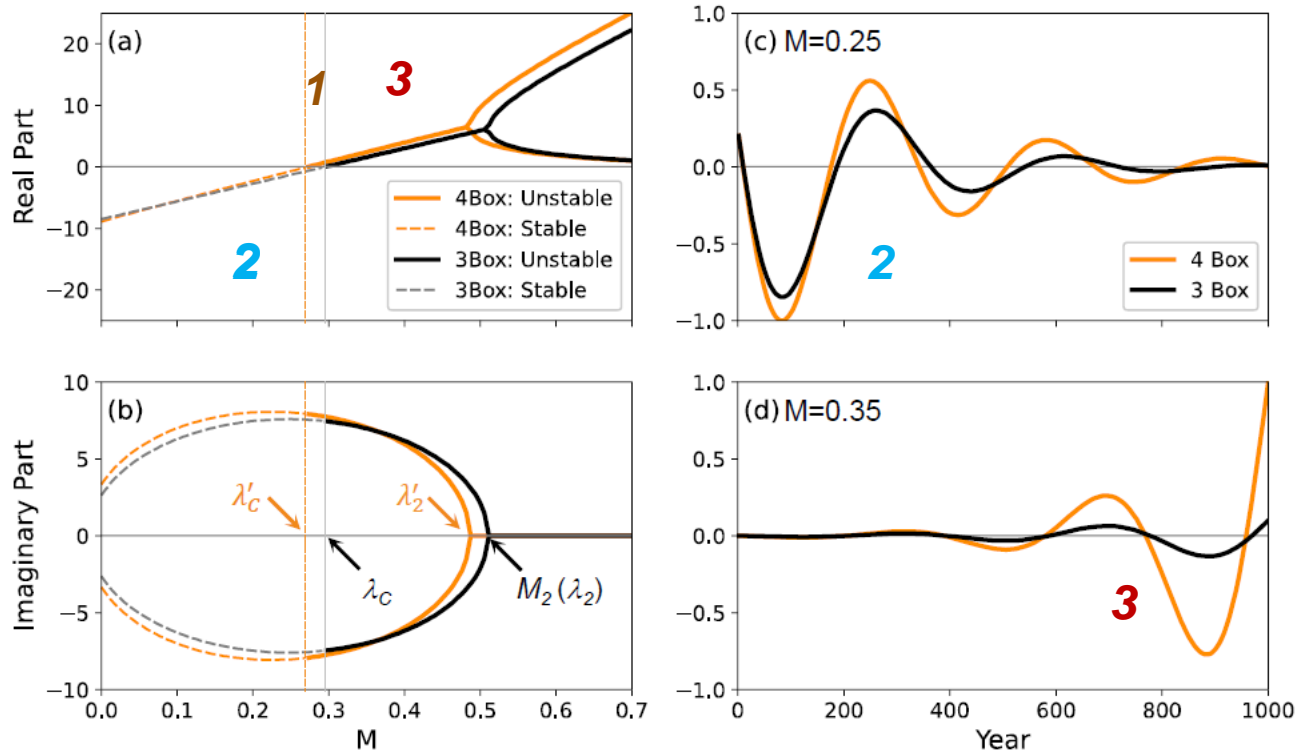
# Self-Sustained Oscillation with Convection



Li and Yang (2021)



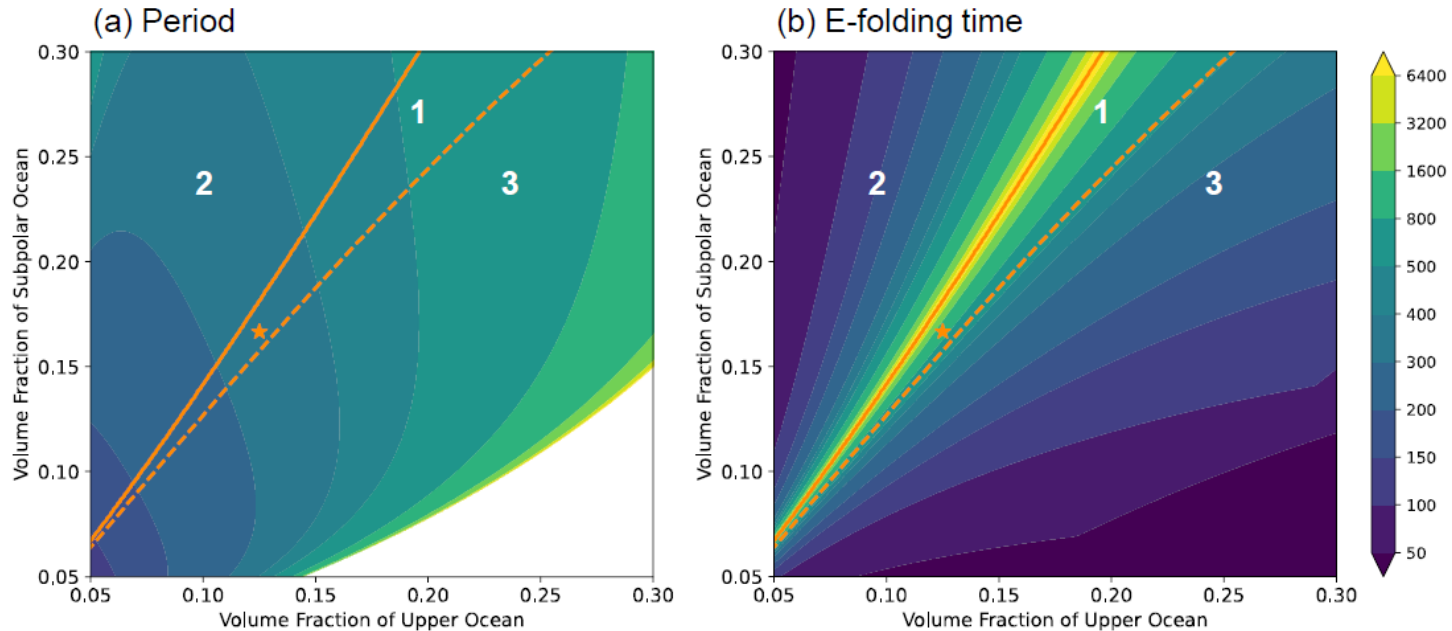
# Self-Sustained Oscillation with Convection



Li and Yang (2021)

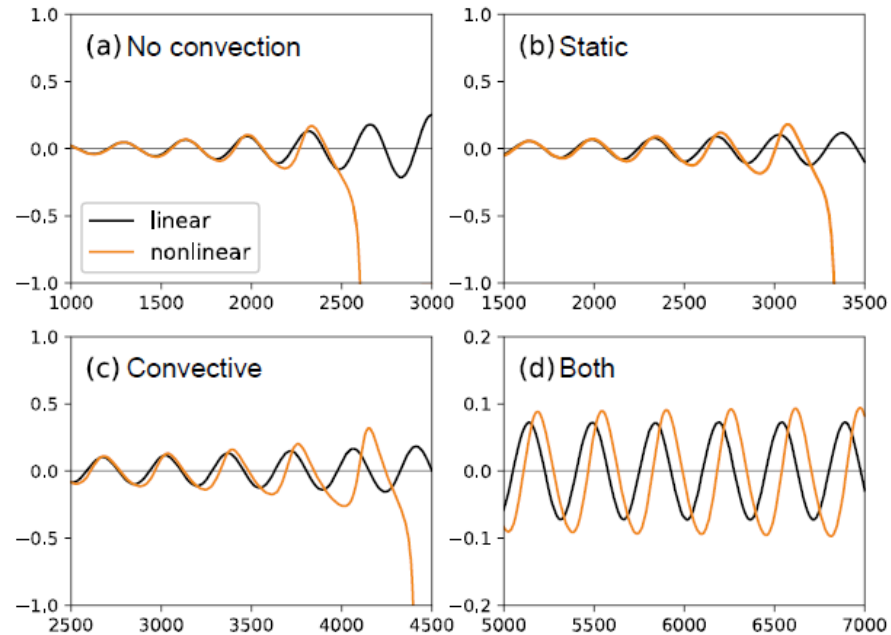


# Self-Sustained Oscillation in Depth Space



Li and Yang (2021)

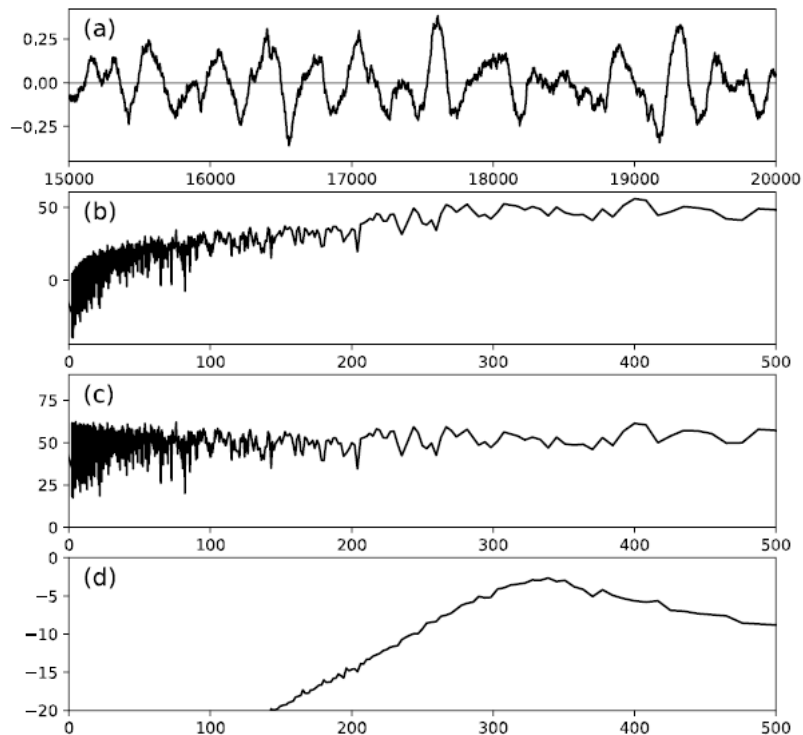
# Nonlinearity: Self-Sustained Oscillation



Li and Yang (2021)



# Self-Sustained Oscillation under Stochastic Forcing

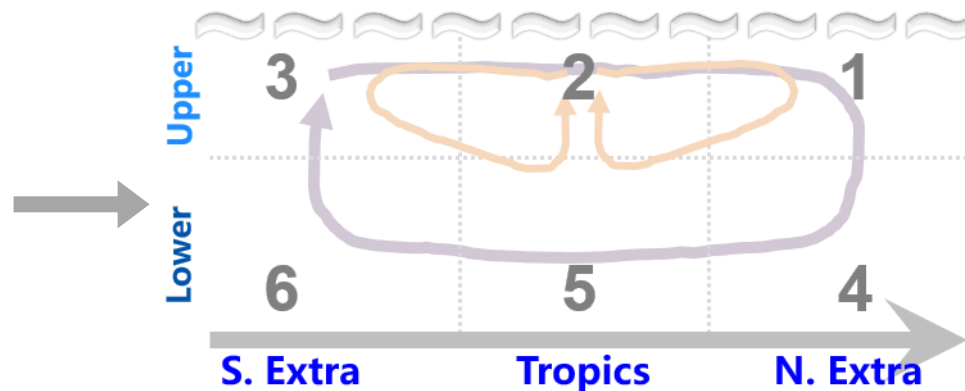


Li and Yang (2021)



# 6-Box Model for Two Hemispheres

## Symmetric WDC and Global THC



6 temperature + 6 salinity = 12 equations

Shi and Yang (2020)

# Eigen Modes in a 6-Box Model

$$\frac{\partial}{\partial t} \begin{pmatrix} T \\ S \end{pmatrix} = M \begin{pmatrix} T \\ S \end{pmatrix}$$

$M$ : 12x12 matrix →  
12 Eigen modes

Two  
oscillation  
modes

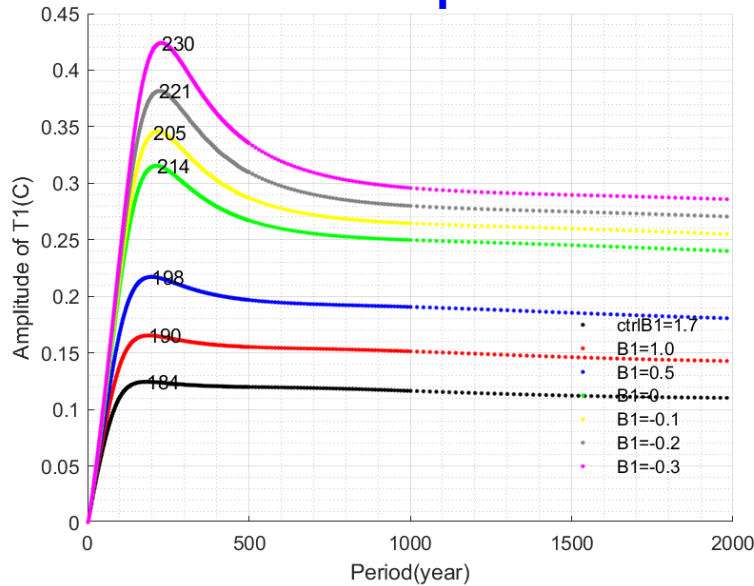
CTRL	B1=0.5	B1=0	B1=-0.3
0.6530	0.8406	1.0160	1.1994
183年	193年	205年	221年
-6.5	-8	-10	-12+92i
-14.3	-14	-13	-12-92i
-35.1	-42	-46	-50
-131.4+202.0i	-187+224i	-246+242i	-310+258i
-131.4-202.0i	-187-224i	-246-242i	-310-258i
-261.9	-376	-508	-674
-3281.3	-4522	-5819	Nan
Nan	Nan	Nan	-7353
-1187+7341i	-1685+11791i	-2242+18925i	-2916+31770i
-1187-7341i	-1685-11791i	-2242-18925i	-2916-31770i
-1044+1543i	-1473+2183i	-1948+2892i	-2522+3745i
-1044-1543i	-1473-2183i	-1948-2892i	-2522-3745i
0.6789	0.6756	0.6754	0.6722

Shi and Yang (2021)

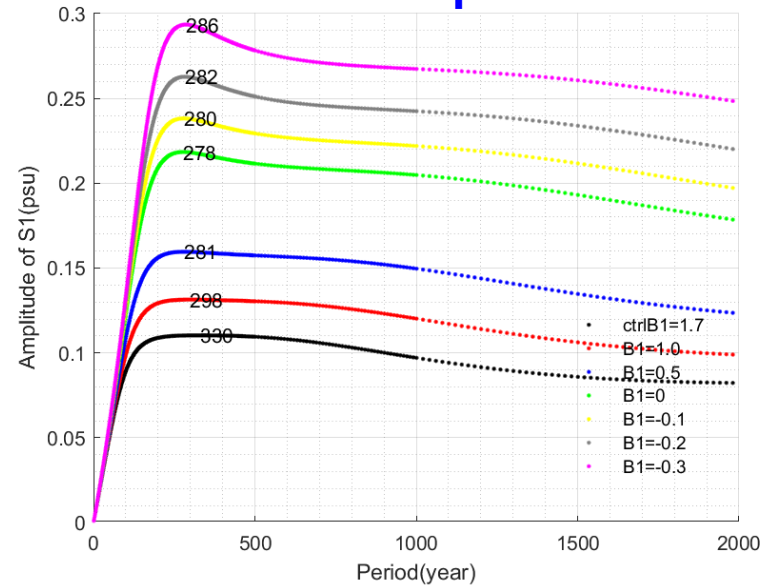
# Multi-Centennial Mode

Can be easily excited by random forcing!

## N. Extratropical T1



## N. Extratropical S1

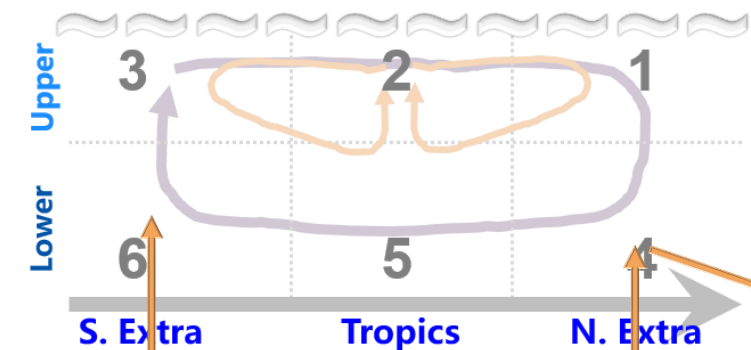


Shi and Yang (2021)



# Millennial Mode in 6-Box Model

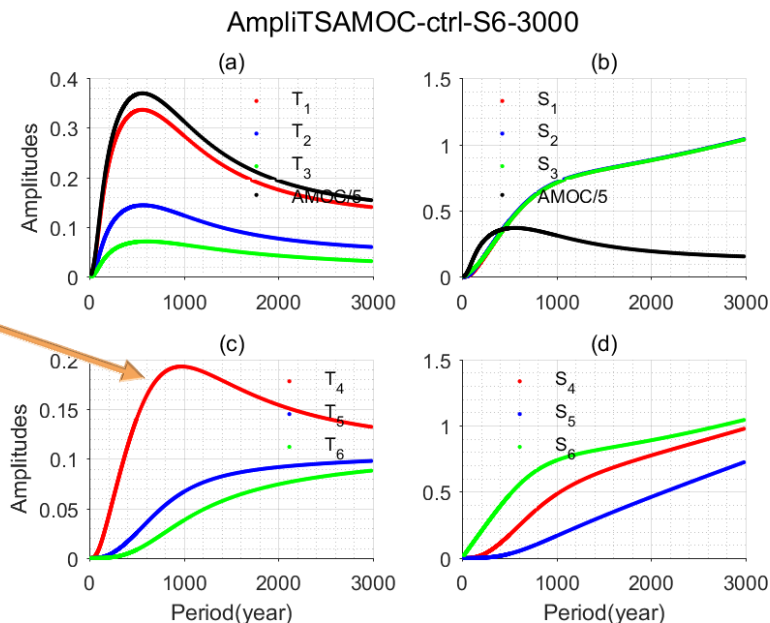
Can be *only* excited by *deep SO* perturbation (??)



Perturbation here

Response here

$T_6 \rightarrow S_4; S_6 \rightarrow T_4$



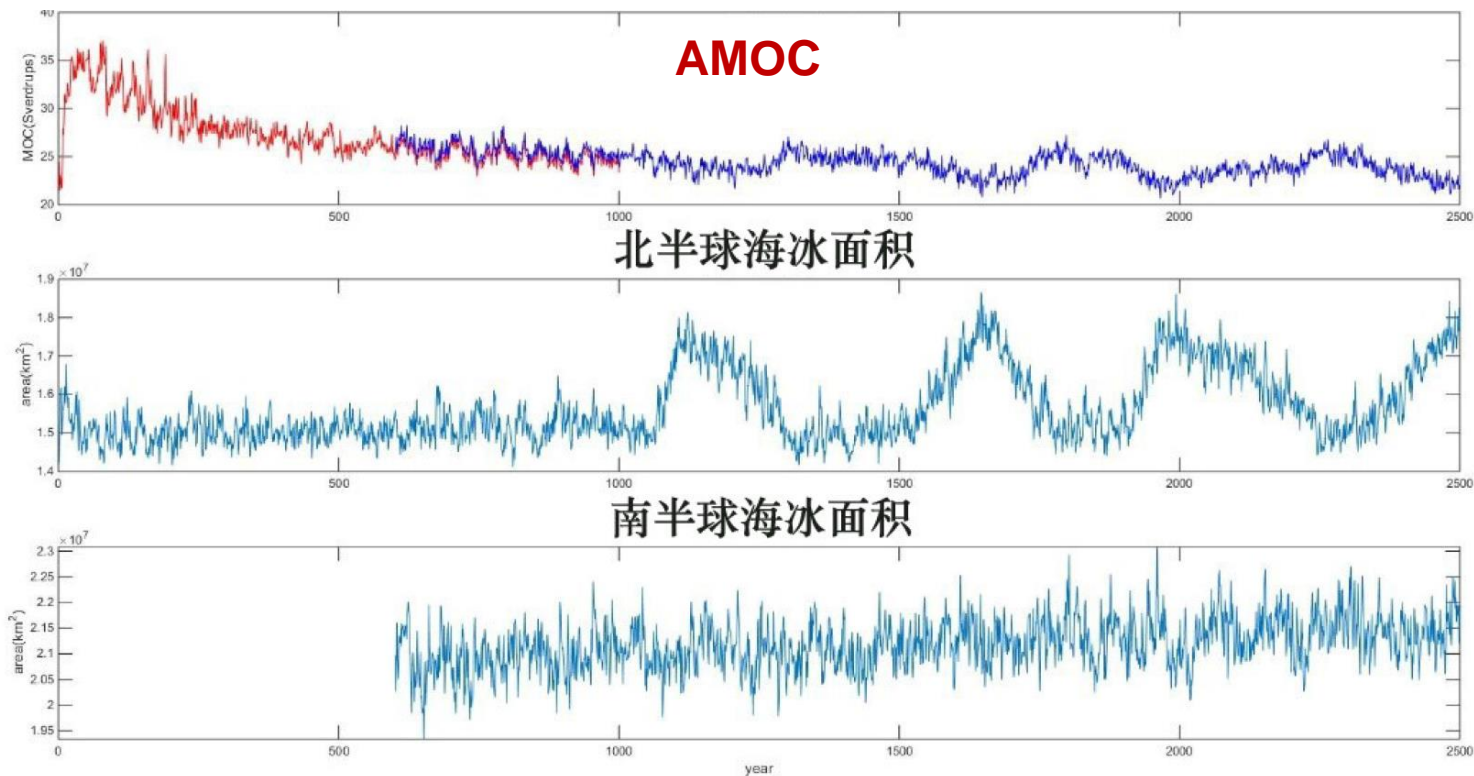
Shi and Yang (2021)

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1. Common Knowledge
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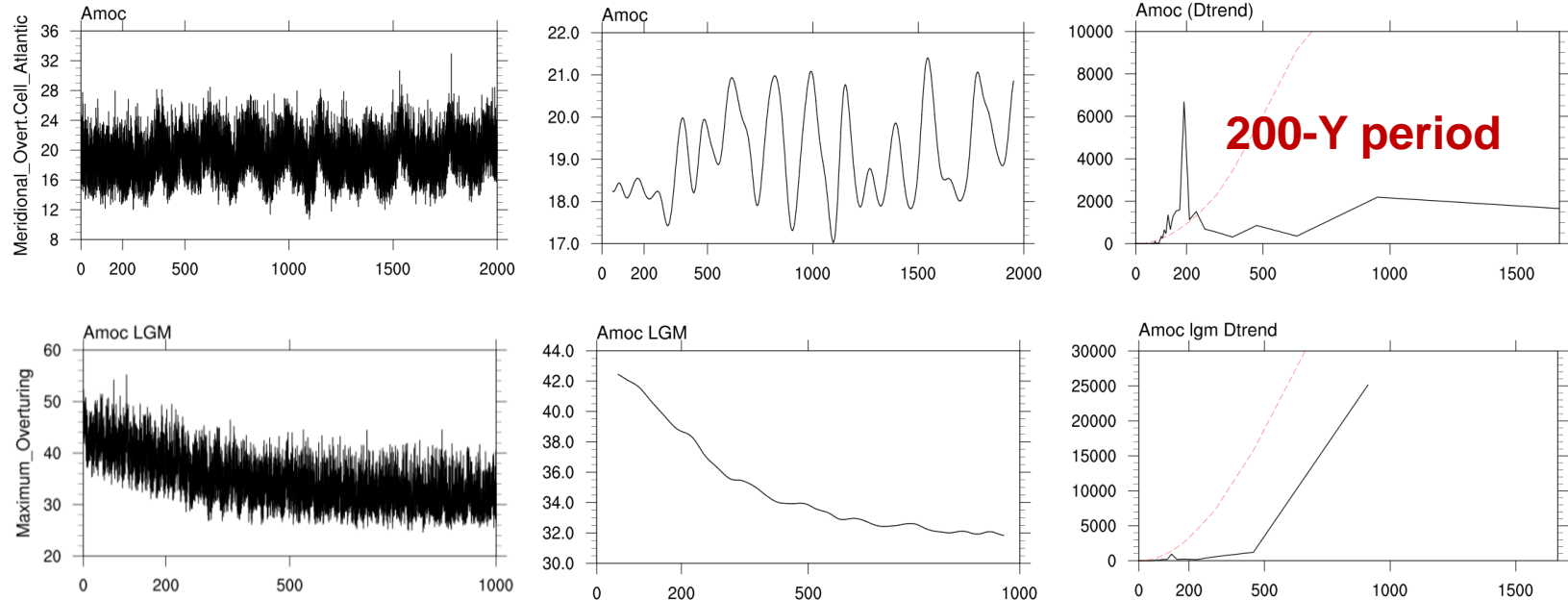
# Centennial Oscillation in CESM2.0

## 2500 Years period PI control experiment



# Centennial Oscillation in EC-Earth3 Model

## 2000 Years period PI control experiment

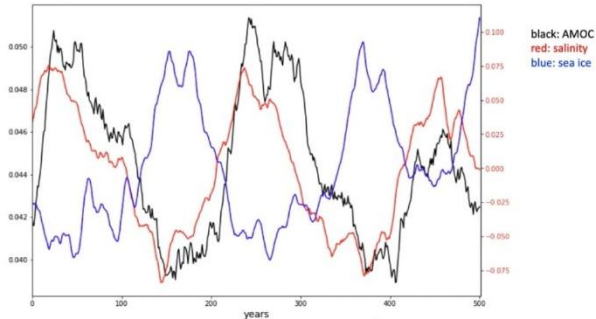


Zhang et al. (2021)



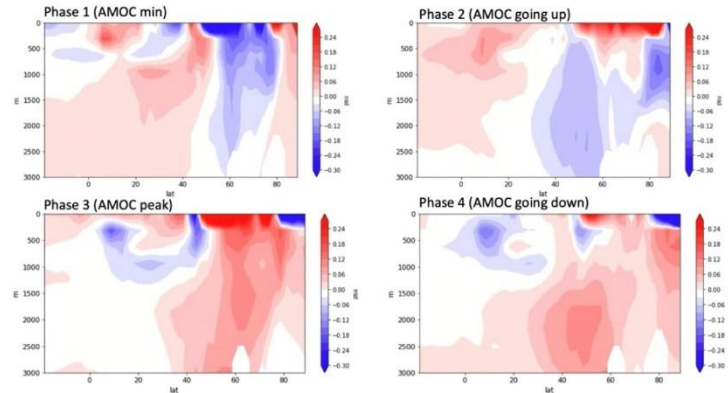
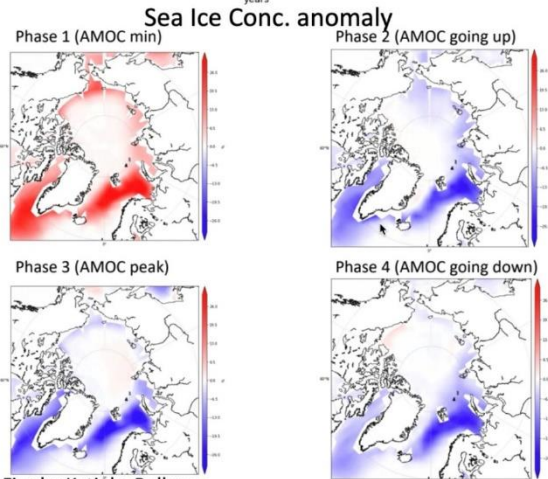


# Exists also in Other Models ...



An example of topic of possibly broader interest: Oscillations in PI EC-Earth simulations

## Salinity anomaly



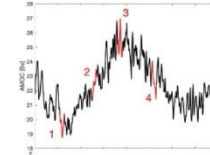
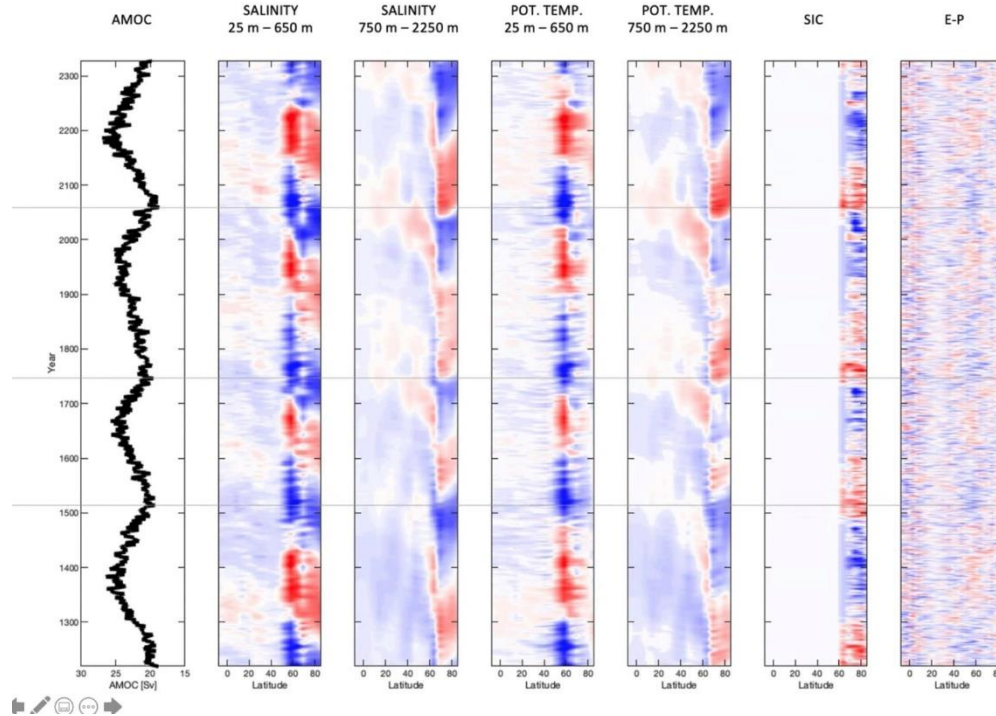
An understanding of these oscillations has implications for:

- better tuning and creation of equilibrated ICs of the model
- Interdecadal variability in EC-Earth
- Paleoclimate and tipping points
- A better understanding of mechanisms associated with AMOC decrease in projections

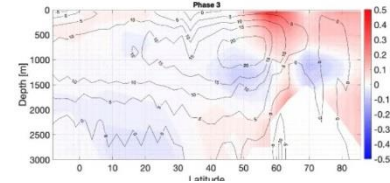
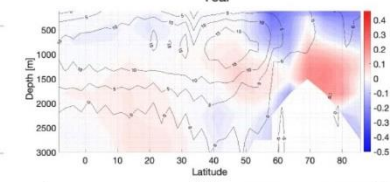
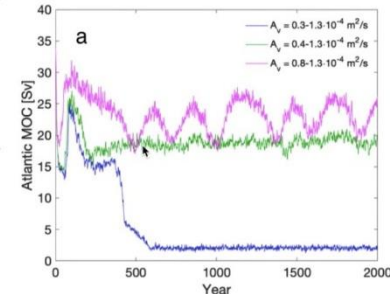
Jost von Hardenberg (2021) Personal communication

# Exists also in Other Models ...

## AMOC oscillations in the coupled PlaSim-LSG EMIC

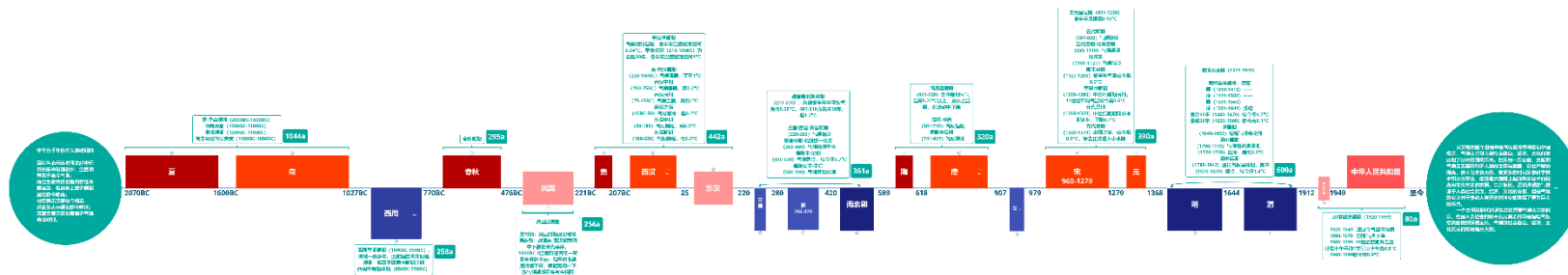
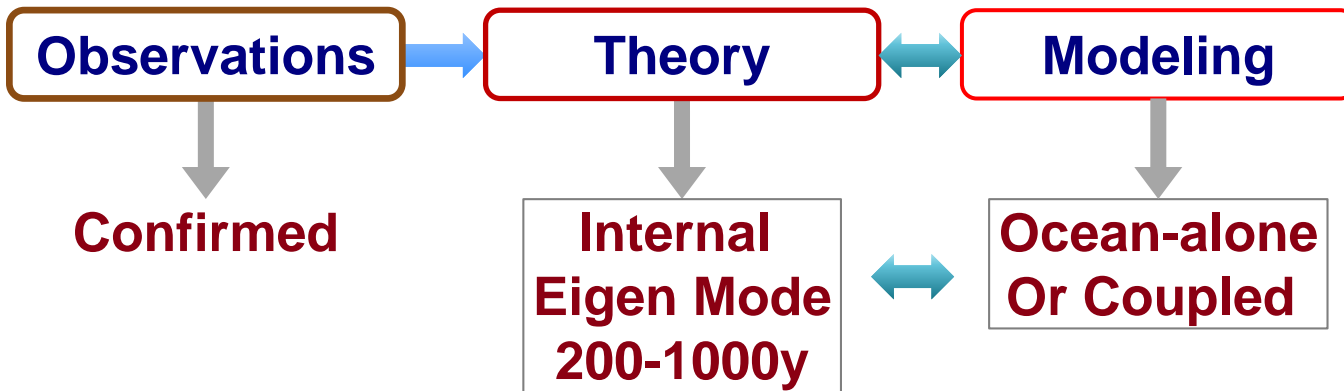


TIPES  
TIPPING POINTS IN THE EARTH SYSTEM



Angeloni et al. (2021)

# Centennial-Millennial Variabilities



# Centennial-Millennial Climate Variability

## A 10-year long way ...

- Eigen Mode: **likely**
- Physics: can be disclosed
- Southern ocean matters for millennial
- Salinity change (freshwater) matters
- Advection-feedback process dominates

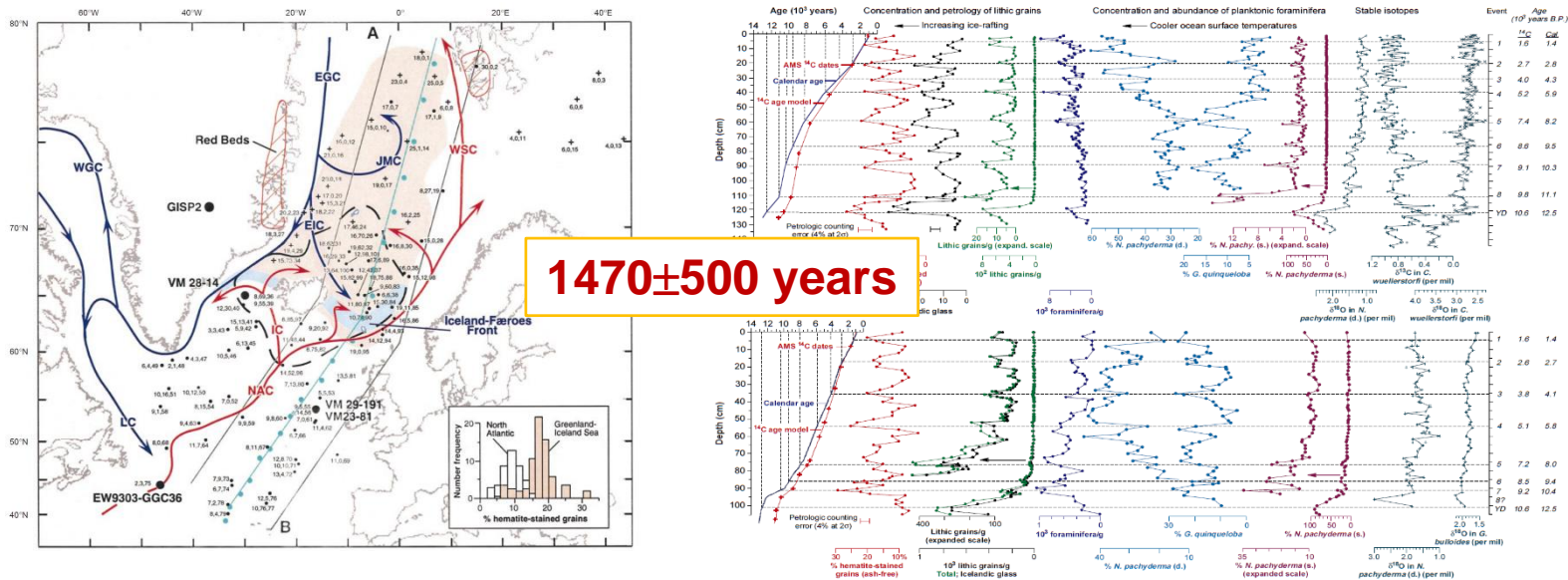


LaCOAS  
北京大学气候与海-气实验室

谢谢

# Millennial Variability- Bond Cycle: ~1500 (?) Years

Bond and Lotti (1995), Science; Bond et al. (1997), Science; Bond et al. (1999) ...

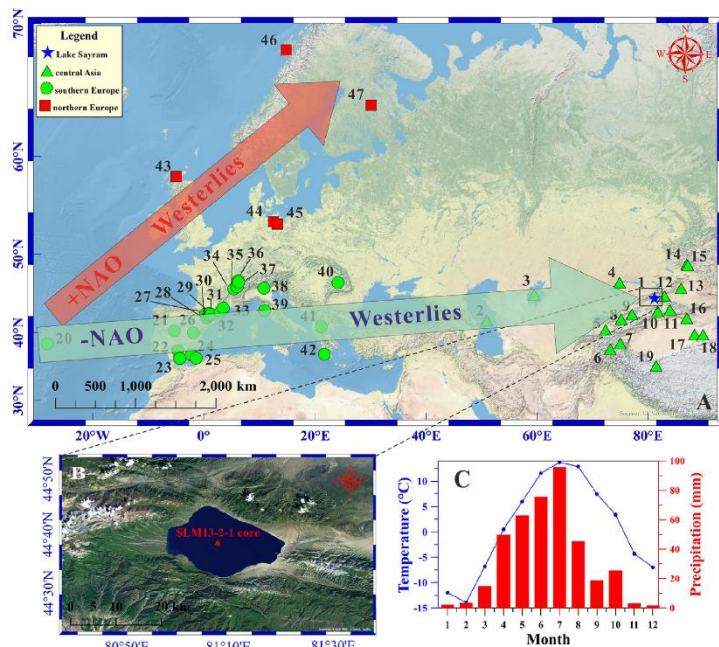


Pacings of the Holocene events and of abrupt climate shifts during the last glaciation statistically the same;  
 The Holocene events the most recent manifestation of a pervasive millennial-scale climate cycle,  
 independent on the glacial-interglacial climate state. Amplification of the cycle during the last glaciation  
 linked to the North Atlantic's thermohaline circulation.





# Millennial Variability- Bond Cycle: ~1500 (?) Years



Lan et al. (2020)

Contents lists available at ScienceDirect

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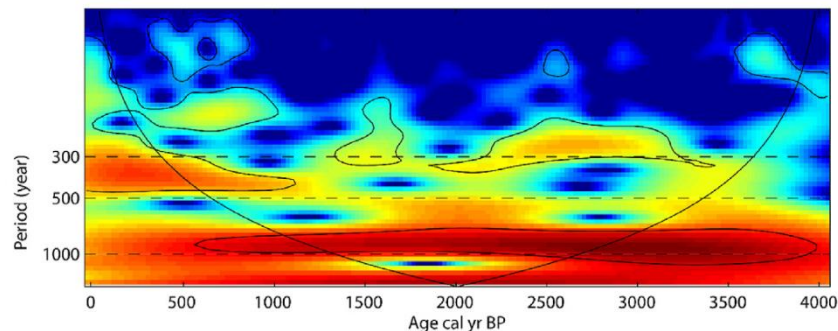
journal homepage: [www.elsevier.com/locate/quascirev](http://www.elsevier.com/locate/quascirev)

QUATERNARY SCIENCE REVIEWS

Check for updates

Late Holocene hydroclimatic variation in central Asia and its response to mid-latitude Westerlies and solar irradiance

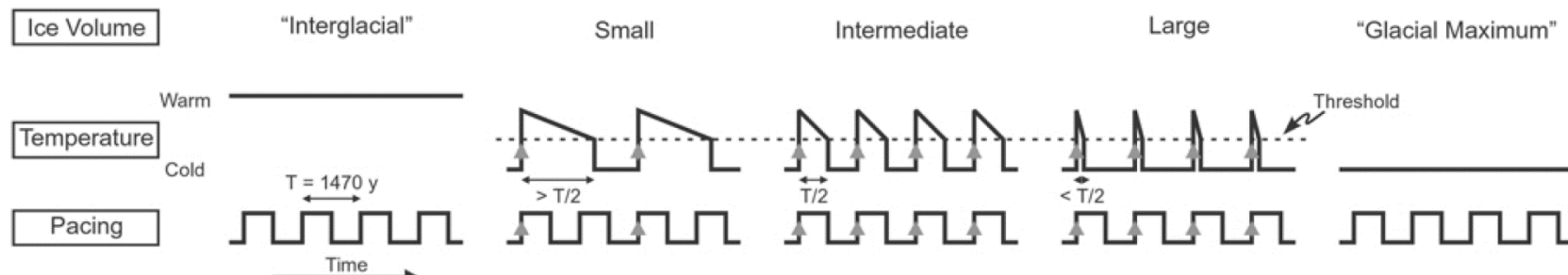
Jianghu Lan <sup>a,b,\*\*</sup>, Jin Zhang <sup>c,\*</sup>, Peng Cheng <sup>a,b</sup>, Xiaolin Ma <sup>a,b</sup>, Li Ai <sup>a,b</sup>, Sakonvan Chawchai <sup>d</sup>, Kang'en Zhou <sup>a,c</sup>, Tianli Wang <sup>a,c</sup>, Keke Yu <sup>f</sup>, Enguo Sheng <sup>g</sup>, Shugang Kang <sup>a,b</sup>, Jingjie Zang <sup>a</sup>, Dongna Yan <sup>a,e</sup>, Yaqin Wang <sup>a</sup>, Liangcheng Tan <sup>a,b,h</sup>, Hai Xu <sup>c,b</sup>



**Fig. 7.** Evolutionary spectral analysis of hydroclimatic variations recorded by  $\delta^{13}\text{C}_{\text{carb}}$  from Lake Sayram over the past 4000 years. Black lines indicate >90% significance levels.

# Millennial Variability- Bond Cycle: ~1500 (?) Years

Dansgaard-Oeschger interstadials → 1470-year climate cycle



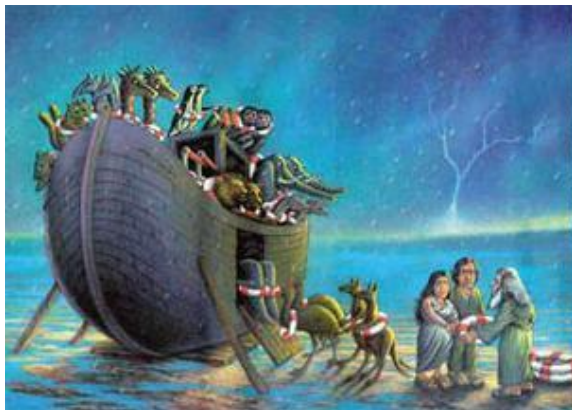
1470-y pacing cycle as function of continental ice volume.

Actual shape of the pacing cycle (bottom row) is unimportant because this signal acts only as trigger (vertical arrowheads) for Dansgaard-Oeschger type temperature fluctuations (center row)

Michael Schulz (2002), GRL

## 人类文明的兴衰史也与百年-千年的气候振荡有密切关系

诺亚方舟



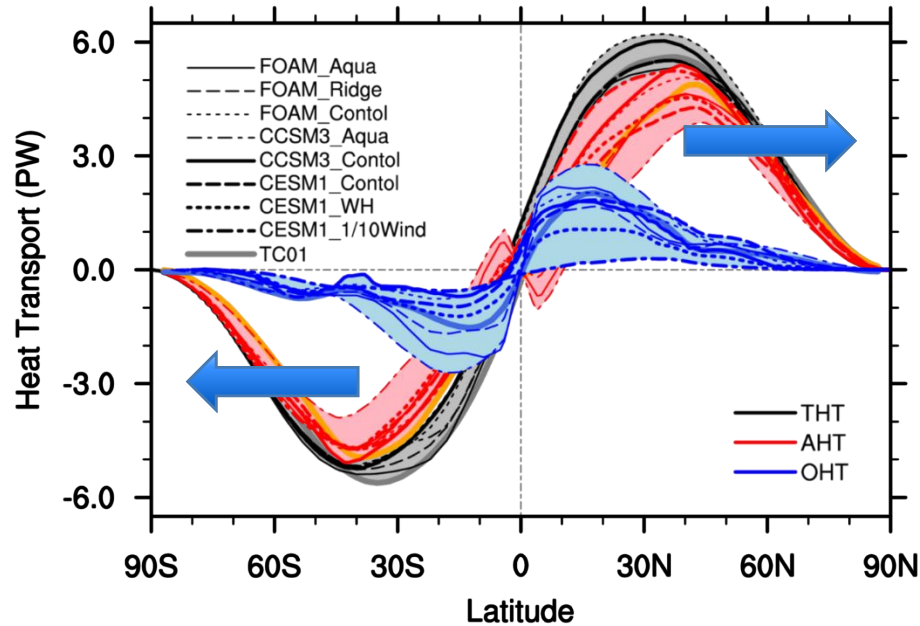
大禹治水

- 王绍武 (2005a) : 公元前2.2-2千年尼罗河文明、两河流域文明及印度河文明等等突然衰落发生在中纬度普遍变冷的气候背景中, 是全新世进入大暖期以来的一次强冷事件
- 王绍武 (2005b) : 洪水→干旱→中华文明的诞生→公元前2070年夏朝建立
- 王绍武和黄建斌 (2006) : 夏朝建立的基础“大禹治水”, 气候干旱可能对“治水”成功产生了影响
- 王绍武等 (2011) : 距今6-4千年的五帝时代: 湿润→干旱; 可能与热盐环流的突然减弱有关



# Bjerknes Compensation (BJC)

## A *Eigen Mode* of Coupled Ocean-Atmosphere System



Yang et al. (2015, 2016, 2017, 2018); Liu et al. (2016, 2019)

# Bjerknes Compensation (BJC)

A *Eigen Mode* of Coupled Ocean-Atmosphere System

**Robust and theoretically derived!**

Climate Shift:

$$C_{R0} \equiv \frac{F'_a}{F'_o} = -\frac{1}{1+B/2\chi}$$

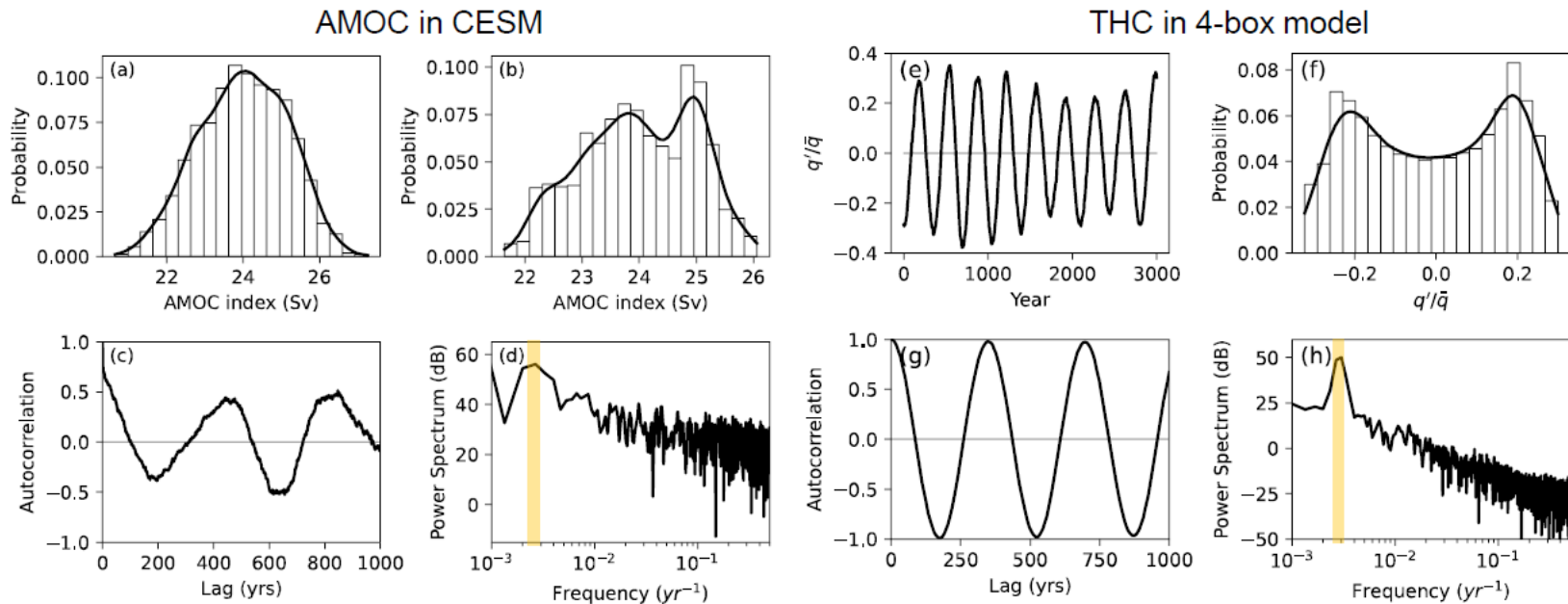
↕  
Climate Variability:

$$C_{Rp} \equiv \frac{F'_a}{F'_o} = r_\delta * C_{R0}; \quad r_\delta \equiv \cos\delta = -\frac{F}{\sqrt{\omega^2+F^2}}$$

$$\omega \rightarrow 0 \Rightarrow r_\delta \rightarrow -1; \quad C_{Rp} \approx C_{R0}$$

Yang et al. (2015, 2016, 2017, 2018); Liu et al. (2016, 2019)

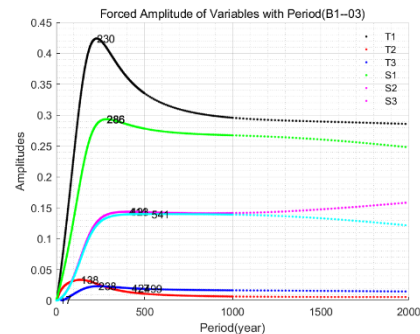
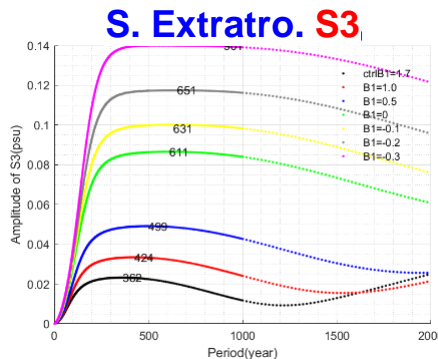
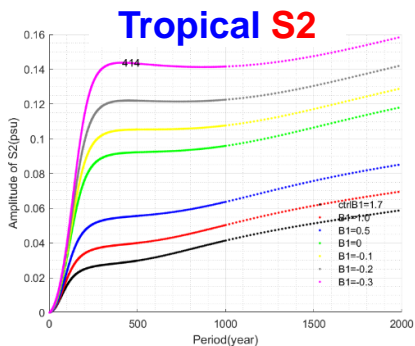
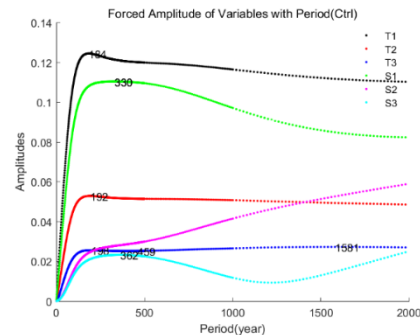
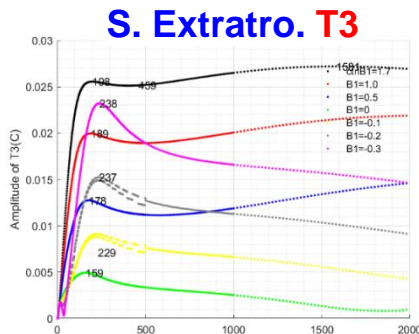
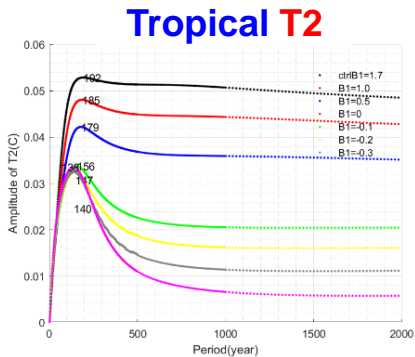
# Self-Sustained Oscillation in CESM?



Li and Yang (2021)

# Multi-Centennial Mode

Can be easily excited by random forcing!



Shi and Yang (2021)



## Features:

- Significant signal in *upper* ocean, *Northern* Hemisphere
- Also in *Tropics*
- Self-sustained / Random forcing

Shi and Yang (2021); Li and Yang (2021)

# Millennial Mode in 6-Box Model

Determined by the *lower ocean depth*

D2=5000	CTRL	D2=3000	D2=2000	D2=1000
0.65	0.65	0.65	0.65	0.70
-6.53	-6.5	-6.53	-6.53	-6.53
-14.26	-14.3	-14.26	-14.2	-14.26
-35.07	-35.1	-35.07	-35.07	-35.07
-131.39+ 202.08i	-131.4+202.0i	-131.45+ 201.87i	-131.59 + 201.06i	-131.27+ 190.70i
-131.39- 202.08i	-131.4-202.0i	-131.45 - 201.87i	-131.59 - 201.06i	-131.27 - 190.70i
-264.70	-261.9	-253.28	-223.25	-140.48
-4101.71	-3281.3	-2461.00	-1640.72	-820.67
nan	Nan	Nan	nan	nan
-1484.22 + 9179.53i	-1187+7341i	-890.56 + 5500.67i	-593.74 + 3656.27i	-296.93 + 1797.81i
-1484.22 - 9179.53i	-1187-7341i	-890.56 - 5500.67i	-593.74- 3656.27i	-296.93 - 1797.81i
-1283.99+ 2027.32i	-1044+1543i	-814.87 + 1075.65i	-612.35 + 658.49i	-437.72+ 335.90i
-1283.99 - 2027.32i	-1044-1543i	-814.87 - 1075.65i	-612.35 - 658.49i	-437.72- 335.90i
0.6333	0.6789	0.7568	0.9299	1.3031

Shi and Yang (2021)